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BOOK OF ABSTRACTS

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Current Trends in Nonlinear Photonics

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There is a long story of the role nonlinearity plays in optics and photonics. From the early experiments in second harmonic generation to the demonstration of optical solitons in fiber optics, the field continues to evolve in several directions. Rather than presenting well developed ideas, in this talk I will attempt to provide examples mostly from recent experiments in need of better models. They will highlight optical phenomena for weakly and for strongly nonlinear effects. Topics I hope to cover include pulses in novel waveguides with leading fourth order dispersion, spatio-temporal dynamics in multi-mode fibers, optical rogue waves and second harmonics filaments.

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Numerical Investigation on Fractional Variational Problems Depending on Indefinite Integrals

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In this paper, we solve the dynamical fractional variational problems depending on indefinite integrals. The Bernstein operational matrices were applied to obtain the approximate analytical solution of this problem. In this way, the model is reduced to an algebraic easily solvable system. The obtained solutions are very accuracy and the method is very efficient and simple in implementation

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Harmonizing The Incomplete Statistical Data on The Example of Institutional Factors' Impact on Pension Systems

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In the article a software program is introduced, which allows to increase the accuracy of the statistical analysis while having incomplete data. The program finds the changing pattern of the data and attempts to fill in the missing cells with some values. It is using linear regression and arithmetic average methods (for now) to fill in the missing cells. A comparison analysis is given, done before the filling the data with the software and after.

The pension market was taken as a subject for that analysis, with the factors impacting it; particularly, in our research we present the econometric study about the role of religion in the development of the pension market. Using the methods of economic regression - correlation analysis, the least squares method we identified the factors greatly influencing on the target features of pension systems. The analysis also shows the importance of such institutional factors as confidence in the government, corruption, and rule of law for the development of the pension market.

Comparing the models and the econometric analysis accuracy with and without using the program for harmonizing the statistical data, we show the efficiency of the program and point the key situations where it will benefit the research, and where will not.

Keywords: Software, statistical data, harmonizing, missing data, econometric research

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A Guaranteed State Estimation of Linear Retarded Neutral Type Systems

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The problem of minimax guaranteed estimation is considered for time-delayed linear systems of neutral type with uncertain input and output disturbances belonging to a ball in a Hilbert space. It is supposed that the initial state of the system is completely unknown. The infinite-dimensional informational sets are introduced in the appropriate Hilbert space and their properties are investigated. Some particular cases are considered and examples are given. The conditions for infinite-dimensional informational sets to be bounded are given as well. This matter is closely related with the notion of complete observability. Criteria for this are discussed. A finite-dimensional approximation scheme for the problem is developed. The theory of closed operators and semigroups serves here as the main mathematical apparatus.

Unlike retarded equations the continuous solutions of neutral type systems have only the bounded variation and are not absolutely continuous functions. Therefore, some special assumptions are made to provide necessary properties of solutions in order that they lay in the suitable Hilbert space.

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The Dynamics of Two Linearly Coupled Goodwin Oscillators

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We study numerically a system of linearly coupled Goodwin oscillators for two regions, which, when isolated, differ in period and amplitude of regional business cycle. This system is described by the following equations [1]:

$$\varepsilon_1\theta_1\frac{d^2y_1}{dt^2}+(\gamma_1-\varphi_1)\frac{dy_1}{dt}+s_1y_1-\theta_1\left(m_1\frac{dy_1}{dt}-m_2\frac{dy_2}{dt}\right)-m_1y_1+m_2y_2=0,$$
$$\varepsilon_2\theta_2\frac{d^2y_2}{dt^2}+(\gamma_2-\varphi_2)\frac{dy_2}{dt}+s_2y_2+\theta_2\left(m_1\frac{dy_1}{dt}-m_2\frac{dy_2}{dt}\right)+m_1y_1-m_2y_2=0.$$

Here $y_i(t)$ = regional income, $\epsilon_i>0$, $\theta_i>0$ = time-lag of the dynamic multiplier and the time-lag between investment decisions and the resulting outlays, s_i = marginal propensity to save, $0\le s_i\le 1$, $\gamma_i=\varepsilon_i+\theta_i s_i$, m_i = marginal propensity to import, $0\le m_i\le 1$, $\varphi_i(x)$ = induced investment function, $\varphi_i(x)\ge 0$; $\varphi_i(0)=0$; $\varphi_i'(0)=r_i>0$; $\varphi(\dot{y})\to -I_{\min i}$ if $x\to-\infty$; $\varphi_i(x)\to I_{\max i}$ if $x\to\infty$, $i=1,2$.

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Using Personalised Virtual Heart Models to Gain Insights into Causes of Sudden Cardiac Death

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Background: Patients with ST-segment elevation myocardial infarction (STEMI) are at an increased risk to lethal ventricular fibrillation (VF). However, an effective risk factor that can accurately identify which patients are most susceptible to VF and would benefit from invasive prophylactic procedures remains elusive.

Objective: To demonstrate that patient specific, virtual heart models can be used to stratify VF risk among STEMI patients. Methods: MRI and clinical follow up data were obtained from 10 patients with first STEMI, 5 reported subsequent VF. Multiscale, biophysically-accurate virtual heart models were developed from MRI of these patients, including the ischemic zone. Tissue within the ischemic zone was modeled with decreased conduction and altered action potential morphology. Arrhythmia risk in the models was assessed by simulating arrhythmia inducibility using programmed electrical stimulation with up to 3 extra-stimuli delivered at the right ventricular apex.

Results: The simulations successfully predicted arrhythmia risk in all 10 patients. The 5 patients with arrhythmia had larger ischemic zones than the control patients (15.2cm^3 *vs* 6.1cm^3). In the arrhythmic patient models, arrhythmia was initiated due to decreased conduction within the ischemic zone that promoted wave break and reentry formation. The reentrant circuits persisted throughout the duration of the simulation (2 seconds) and remained localized within the ischemic zone. The 5 control patient virtual hearts were not inducible for arrhythmia.

Conclusions: Simulations with patient-specific virtual heart models of ischemic hearts may provide a unique opportunity to noninvasively predict arrhythmia risk in patients.

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Analysis of Stochastic Phenomena in Ricker-Type Population Model with Delay

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A problem of the analysis of the stochastic persistence regimes and the noise-induced extinction in population models is considered. For this study, we suggest a new mathematical technique combining an analysis of the geometry of attractors and their stochastic sensitivity. In the present talk, our approach is demonstrated for the conceptual discrete Ricker-type population model with Allee effect and delay. Persistence zones of the initial deterministic model are constructed on the base of the bifurcation analysis. It is shown that the random environmental noise can contract, and even destroy these persistence zones. A parametric analysis of the probabilistic mechanism of the noise-induced extinction is carried out with the help of the unified approach based on the sensitivity functions technique and confidence domains method.

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Controlling Stochastic Sensitivity by The Dynamic Regulators

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A problem of the control of stochastically forced equilibria in nonlinear dynamic systems with incomplete information is considered. Our approach is based on the idea of the synthesis of the required stochastic sensitivity for the equilibrium. We consider an important, from engineering point of view, case when the system states are observed partially, and these observations contain random noises. To solve this problem, a dynamic regulator consisting of the feedback and filter is constructed. Mathematically, the problem of the synthesis of the assigned stochastic sensitivity is reduced to the solution of the matrix equations for coefficients of the regulator. Problems of the controllability and attainability are discussed. The effectiveness of the proposed approach is demonstrated on the examples.

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The Development of Novel Cardiac Arrhythmia Electrotherapies Using Computational Cardiac Electrophysiology

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Ventricular fibrillation (VF) is a lethal cardiac arrhythmia that causes sudden cardiac arrest, which takes as many lives in developed countries as cancer, stroke, and AIDS combined. The most effective therapy for terminating VF before death ensues is an aggressively strong far-field electric shock delivered by an implantable cardioverter-defibrillator. Consequently, this can cause severe pain, tissue damage, and increased mortality. As an alternative, I am developing a painless damage-free electrotherapy for terminating VF called WAYLESS: Wide-Area Yielding Low-Energy Surface Stimulation. The basic strategy of WAYLESS is to lower the energy for terminating VF below the human pain and tissue damage thresholds, and to do so with low-energy DC surface stimulation administered via electrodes strategically placed over wide-areas of the ventricles. Accordingly, I use computational cardiac electrophysiology, which combines computer science, applied mathematics, modeling/simulation, and clinical/experimental research to study cardiac function and electrical disorders from the cell to organ level. Thus, WAYLESS stimulation protocols and electrode configurations are first developed in a finite element model of the human ventricles that simulates clinical VF, and are then validated with animal experimentation using a minimal number of large mammalian hearts. From this ethical and cost-effective approach, computer simulations predict WAYLESS with single short-duration low-energy DC pulses, administered with uniformly spaced endocardial line electrodes, to be the most effective at terminating VF at energies $< 1/10$ the human pain threshold of 0.1J, and $< 1/1,500$ the cardiac tissue damage threshold of 15J. Preliminary animal studies for WAYLESS with these line electrode configurations placed on sheep and swine ventricles reveal it is feasible for terminating VF in large mammals. In conclusion, WAYLESS with line electrodes can effectively terminate VF, thus paving the way for a less damaging, pain-free alternative to traditional defibrillation electrotherapies.

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Maximizing the Biogas Production of a Delay Bioreactor Model via Extremum Seeking

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In this talk we investigate a bioreactor model for wastewater treatment by anaerobic digestion. The model equations involve discrete delays, describing the time delay in nutrient conversion to viable biomass. Using a properly chosen admissible value for the dilution rate D we prove the global convergence of the solutions towards an equilibrium point, corresponding to D. We also apply a numerical model-based extremum seeking algorithm for stabilizing the system towards the equilibrium point in which the maximum biogas (methane) flow rate is reached. Numerical simulations are included to confirm the theoretical results. The simulations are implemented in the Web simulation platform SmoWeb.

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Network Models for Solving the Problem of Multicriterial Adaptive Optimization of Investment Projects Management with Several Acceptable Technologies

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This paper discusses the problem of multicriterial adaptive optimization the management of investment projects in the presence of several acceptable technologies for their realization. On the basis of network modeling in this paper proposed a new economic and mathematical model and a method for solving the problem of multicriterial adaptive optimization the management of the process of realization

of investment projects in the presence of several technologies. Network economic and mathematical modeling allows you to determine the optimal time and calendar schedule for the implementation of the investment project and serves as an instrument to increase the economic potential and competitiveness of the enterprise. In the paper, on a meaningful practical example, the processes of forming network models are shown, including the definition of the sequence of actions of a particular investment projecting process, the network-based work schedules corresponding to models are constructed. The calculation of the parameters of network models is carried out. Optimal (critical) paths have been formed and the optimal time for implementing the chosen technologies of the investment project has been calculated. It also shows the selection of the optimal technology from a set of possible technologies for project implementation, taking into account the time and cost of the work. The proposed model and method for solving the problem of managing investment projects can serve as a basis for the development, creation and application of appropriate computer information systems to support the adoption of managerial decisions by business people.

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Modeling of Wave Processes in a Blocky Medium with Fluid-Saturated Porous Interlayers

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Wave processes in a 2D blocky medium are under investigation. Considered continuum consists of rectangular elastic blocks divided by fluid-saturated porous interlayers. Blocks behavior is defined by equations of a homogeneous isotropic elastic medium. To describe dynamic processes in interlayers, a modification of the porous-flow Biot model is used. Governing equations for the porous skeleton are based on Hooke's law, while Darcy's filtration is applied to define the fluid flow. In order to analyze the fluid behavior in nodes between blocks a hydrodynamic analogue of Kirchhoff's law is used.

Further research allows modifying the model proposed earlier. To take pore collapsing effect into account, a specific rheological scheme is suggested. According to this scheme, under approaching some strain level the pore collapses so that the fluid within the pore is occupied. Viscoelastic properties of the skeleton are added into the model by means of the Pointing-Thomson scheme that combines both momentary and delayed elasticity. It was shown that the modified model is thermodynamically consistent.

Computational algorithm is developed for the numerical implementation of presented models. Algorithm is based on a two-cyclic splitting by spatial variables. For the blocks equations Godunov's gap decay scheme is used; for the interlayers equations a numerical method, based on the non-dissipative Ivanov's scheme is applied. Parallel software is designed for analyzing stresses and velocity fields in a 2D blocky medium. Comparative study of the model with elastic interlayers and the model with a fluid-saturated porous material was carried out. It was shown that the latter model preserves isotropic properties of a medium longer than the former model, as the interlayer thickness increases.

Acknowledgement. This work was supported by the Complex Fundamental Research Program no. II.2P "Integration and Development" of SB RAS (project no. 0356-2016-0728).

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Solitary-Wave Solution of Flat Surface Internal Geophysical Waves with Vorticity

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A rotational flat surface system consisting of 2 media with an internal wave and depth dependent current is presented. The Hamiltonian of the system is derived and the dynamics are discussed. A long wave regime is then considered and extended to produce a KdV approximation. Finally, a solitary-wave solution is obtained.

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Numerical Simulation of Pulsed Pirani Sensor Mems

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The transient heat transfer process is studied in rarefied gas confined between two stationary concentric cylinders. The inner cylinder (filament) is subjected to a periodically heating-cooling cycle. The energy transfer is modeled with continuous model based on Navier-Stokes Fourier equations of motion and energy transfer and with a statistical DSMC model. Numerical results for the temperature, thermodynamic pressure and pressure difference between thermodynamic pressure and radial stress tensor component are obtained for different circular frequencies of heating cooling cycle of filament and for different filament radii. The pressure variation at the end of any local heating stage of heating-cooling cycle is close to the value of equilibrium thermodynamic pressure. The results are applicable in designing the pulsed Pirani sensors.

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The Semi-Lagrangian Method for the Navier-Stokes Problem for Viscous Incompressible Fluid

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In the present talk, the two-dimensional time-dependent Navier-Stokes equations are considered for a viscous incompressible fluid in a channel. At the inlet boundary, a flow is given; besides, we impose the no-slip condition at the rigid boundary and the modified “do nothing” boundary condition at the outlet boundary.

To construct a discrete problem, we use the conforming finite element method in the combination with a semi-Lagrangian approach. For the discretization of the Stokes operator of the problem, we use the Taylor-Hood finite elements on rectangles: the biquadratic elements for the velocity components and the bilinear elements for the pressure. These elements satisfy the Ladyzhenskaya-Babuska-Brezzi condition that ensures the pressure stability.

To discretize the nonlinear operator of the transport derivative of the problem, we apply the semi-Lagrangian approximation. In this case, the transport derivative is approximated along a trajectory backward in time. In this approach, special techniques for linearizing the problem are not required. We use several iteration with respect to nonlinearity for diagonal terms only.

To overcome the lack of mass conservation of the classical semi-Lagrangian method, we propose its conservative version. To guarantee the mass conservation and the stability in the mean-square norm, we use the discrete equation of the local integral balance between two neighboring time levels.

As a result of this combined approach, the stationary problem with a self-ajoint operator is obtained on each time level. A solution of the problem is calculated by the modified gradient method for a saddle point system ("Inexact Uzawa conjugate gradient method for the Stokes problem for incompressible fluid," *AIP CP* **1773**, 2016, paper 100002, 9p).

Acknowledgement. The work is supported by Project 17-01-00270 of RFBR.

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Evolutionary Dynamics in Cancer Cell Populations and Optimal Control of Targeted Chemotherapy Delivery

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We approach the controlled evolutionary dynamics in cancer cell populations by controls of targeted chemotherapy delivery. The reference system is defined by a sample of cancer cells characterized by heterogeneous genotypic-phenotypic profiles corresponding to different level of proliferative potential. The sample is exposed to the action of Targeted Chemotherapeutic Agents (TCA's, in the sequel), which selectively kill cancer cells, according to their genotypic-phenotypic profiles. The dynamics of the reference system is described by the ODE system given hereafter

$$\begin{cases} \dfrac{d}{dt}\varrho(t) = \left(r - d\varrho(t) - \mu g(t)\right)\varrho(t)\,, \\ \dfrac{d}{dt}g(t) = G(t) - \left(\lambda + \mu\varrho(t)\right)g(t)\,, \end{cases}$$

where ϱ represents the cancer cell population density, g is TCA concentration, and d, r, λ and μ are positive constants. Cancer growth and chemo-therapeutic drugs inoculation imply a systemic cost for the human body. As a result, the optimal control problem of targeted chemotherapy delivery can be tackled by introducing a proper functional embodying this cost, say $J[G, \varrho]$, and looking for the existence of an optimal infusion rate G^*, allowing the minimization of such a functional over $[0,T]$. We define the objective cost functional as $J[G,\varrho] := J_1[G]+J_2[\varrho]$, where functionals J_1 and J_2 measure, respectively, the systemic costs to the body introduced by the proliferation of tumor cells and the inoculation of chemotherapeutic agents. Among the possible definitions for $J_1[G]$ and $J_2[\varrho]$, we focus on the ones given hereafter

$$J_1[G] = \gamma \int_0^T G^2(t)\, dt\,, \qquad J_2[\varrho] = \int_0^T \varrho(t)\, dt\,,$$

which measure, respectively, the concentrations of tumor cells and cytotoxic agents inside the system over the time interval $[0,T]$. It is worth noting that the quadratic dependence on function G accounts for the nonlinear dependence of side effects on the infusion rate of chemotherapeutic agents. Finally, coefficient $\gamma > 0$ includes a measure of toxicity of chemotherapies to the human body; the higher the weight, the greater the toxicity. The numerical solution of the optimal control problem corresponding to various scenarios is presented.

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Accounting Services for Heterogeneous Computing Resources

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The accounting platform is a web-service based system for collection and analysis of accounting data from different infrastructure resources like HPC, Cloud and storage systems. The platform has two major components – backend API services along with different data publishers and a client web UI module for visualisation and operations. The backend API is designed to gather information from different job management systems, cloud vendors and storage providers and use microservice architecture. The web UI module is written in python, javascript and has integrated SAML login module for user authentication and authorization. It is capable of visualising the gathered data in dynamic OLAP style and supports standard export formats like csv and Excel. Through the accounting platform it is possible to obtain full view of the usage patterns of an integrated electronic infrastructure and to see from one point all information about the different resources comprising the hybrid computing and data infrastructure.

Acknowledgments. This work is supported by EC programme HORIZON2020, Grant Agreement No. 675121, VI-SEEM project and by the National Science Fund of Bulgaria under Grant DFNI-I02/8.

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Exact Results for the Casimir Force in a Model with Neumann-infinity Boundary Conditions

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We consider Ginsburg-Landau model of a system with strongly adsorbing competing surfaces – say, a binary liquid mixture in which one of the bounding surfaces strongly prefers one the components of the system with the second surface preferring the other one. We focus on a system with a plain geometry. We study the behavior of the thermodynamic Casimir force in such a system close to the bulk critical point of the infinite system. For such system, it turns out that it is possible to obtain directly the Casimir force in a manner, that does not require the knowledge of the behavior of the order parameter profile and which is possible to be obtained for the case when both the role of the temperature and the ordering external field is taken into account. This result compliments the exactly known solution for the problem when the field is absent and is in excellent agreement with the numerically known results for the force for the case when the temperature has been fixed to its critical value. In the current article, we present data for the behavior of the Casimir force for some fixed values of the temperature and the ordering external field.

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Numerical Studies of a Fixed Point Iteration Approach for Approximate Deconvolution Models

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Recently, the author proposed in a Nonlinear Analysis RWA paper a new interpretation of the approximate deconvolution turbulence models(ADM). Finite

element schemes based on this interpretation of the ADM lead to nonlinear algebraic systems. Such systems can be solved using fixed point iteration methods. We present herein a numerical investigation of the convergence of such a fixed point iteration method and several numerical tests of it on benchmark problems.

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Numerical Solution of The PTC Thermistor Problem with a Modified Electrical Conductivity

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This paper presents the numerical solution of the one-dimensional PTC thermistor with a modified electrical conductivity. The partial differential equation model is solved using the method of lines. Results are presented to show temperature evolutions and time dependent temperature response in all the phases. Results obtained when compared with exact steady state solution indicate that the method is suitable for the solution of the PTC thermistor problem.

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PIC-Simulation of The Electron Beam Interaction with Modulated Density Plasma

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The work is devoted to the numerical modeling of the interaction of the relativistic electron beam with plasma and generation of high-frequency electromagnetic radiation. Studies of these phenomena are relevant to various physical problems such as the generation of the radiation in solar radio splashes, turbulent plasma heating in open magnetic traps, fast ignition in inertial confinement fusion and formation of collisionless shocks in astrophysics. The two-dimensional model based on the PIC-method (the particle-in-cell method) for solving the Vlasov equation and FDTV scheme for electromagnetic fields, which includes electron beam injection into the magnetized plasma and open boundary conditions, is presented. The electron beam enters into plasma with the modulated density along the magnetic field. The algorithm of parallel computing was developed. Series of numerical experiments for different background plasma parameters, beam and magnetic field have been performed using modern computer systems with massively parallel architecture. Mechanism of generation of terahertz electromagnetic radiation was investigated. The parameters of numerical experiments were close with the parameters of laboratory experiments on GOL-3 facility (BINP SB RAS, Novosibirsk, Russia).

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Some Adequate Discrete Models in Biomathematics

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In biology many phenomena can be described by differential equations.Typically these continuous models cannot be solved analytically,therefore, by using some numerical method, we construct discrete models. These models should reflect the biologically motivated basic qualitative properties of the original phenomena. Our aim is the analysis of these properties for different discrete models. In our talk we will consider discrete models of different processes, namely, the time-space-dependent epidemic propagation (SIS, SIR models), the Lotka-Volterra predator-pray models, and also the discrete models for the Easter Island phenomena. We give sufficient conditions for the disretization parameters in the discrete models under which the models possess the main characteristic properties. We illustrate our results with numerical examples.

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Biofluidmechanics of Reproduction

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From fertilization to birth, successful mammalian reproduction relies on interactions of elastic structures with a fluid environment. Sperm flagella must move through cervical mucus to the uterus and into the oviduct, where fertilization occurs. In fact, some sperm may adhere to oviductal epithelia, and must change their pattern of oscillation to escape. In addition, coordinated beating of oviductal cilia also drives the flow. Sperm-egg penetration, transport of the fertilized ovum from the oviduct to its implantation in the uterus and, indeed, birth itself are rich examples of elasto-hydrodynamic coupling.

We will discuss successes and challenges in the mathematical and computational modeling of the biofluids of reproduction.

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Mathematical Modeling of the Heat Transfer during Pyrolysis Process Used for End-of-Life Tires Treatment

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End-of-Life tires are waste that practically does not decompose in nature. It is advisable this waste to be processed. Burning tires outdoor leads to there lease of a large number of dangerous toxic and carcinogenic organic substances into the atmosphere. Tires ejected or buried under ground decompose in natural conditions for more than 100 years. The contact of these decomposing tires with rainwater and groundwater leads to the formation of organic toxins and carcinogenic chemical compounds. That is why, since the beginning of 2003, the European Union has adopted a decision banning the incineration and burial of tires under the ground.

Every year, over 50 million discarded tires are produced globally, which definitely creates environmental problems. For their processing, it is necessary to create technologies that protect nature and, on the other hand, to obtain energy and raw materials. One of the possible methods for environmentally sound treatment of End-of-life Tires is the process pyrolysis.

Pyrolysis is a chemical process that takes place without the presence of oxygen. Pyrolysis differs from the combustion process due to the lack of waste products. Pyrolysis of End-of-Life tires is a complex process of thermal destruction. It is conducted in chemical reactors where the tires are heated to a certain temperature under which they are decomposed into several fractions – liquid, solid and gaseous. After the initial heating, the process is continued for a certain period of time, after which the reactor is allowed to cool. The liquid and solid products of pyrolysis are then removed and thus a pyrolysis cycle is completed.

The pyrolysis process is 3D and non-stationary and because of this it is very complicated for modeling. Mathematical modeling of the heat transfer during the pyrolysis process used for the treatment of the End-of-Life tires is presented in this paper. A hierarchy of 2D models for the temperature which describe the non-stationary heat transfer in such a pyrolysis station is created. An algorithm for solving the model equations, based on MATLAB software is developed. The results for the temperature for some characteristic periods of operation of pyrolysis station and for different initial heating are presented and commented in the paper.

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Computation of Seismic Wave Fieldkinematics in a Three-Dimensional Heterogeneous Isotropic Medium

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The propagation of seismic waves in three-dimensional heterogeneous isotropic structures is a complicated process. 3-D wave propagation problems are of practical interest in seismology and seismic exploration. In addition, computed rays can be used as a framework for applications of various more sophisticated methods.

The goal of this study is to develop algorithms and programs to compute wave travel times and directions at the nodes of a given 3-D grid. We consider a velocity model that can be described analytically or presented by defining velocity values at the nodes of a 3-D regular point grid. In the latter case,the velocity model is smoothed by a spatial low-pass filtering. An algorithm of 3-D shooting is developed. The algorithm can be used in case of a point source and more generally for an initial wave front position defined as a parametric surface $r = r(u, v)$. In the latter case,the initial points of rays on this surface are not known beforehand.In the case of a source point, the ray direction is determined by the variables u, v. We consider equations for partial derivatives with respect to the variables u, v together with the differential ray system. The entire system is solved by a fifth-order Runge-Kutta method with step-size and precision control. With known derivatives at ray points we can use Newton's iterative method, which guarantees quadratic convergence of the shooting process. Moreover,with these derivatives we can calculate geometrical spreading and thus the amplitudes.

Different types of velocity structures such as homogeneous, gradient, *etc.* and different types of surface $r = r(u, v)$ are considered. The results of numerical experiments are presented.

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Reduced Dimension Modeling of Recruitment/Derecruitment Dynamics in The Lung

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Introduction: Acute lung injury (ALI) is an onset form of non-cardiogenic pulmonary edema that can be triggered by local or systemic inflammation. ALI and its more severe form known as acute respiratory distress syndrome (ARDS) can lead to widespread alveolar collapse and small airway closure. The primary method of treating ALI/ARDS is the use of mechanical ventilation, yet this can lead to further damage to the lung known as ventilator-associated lung injury (VALI). This injury is thought to arise from two mechanisms that include alveolar over-distension during peak inspiration (barotrauma) and the repetitive recruitment of airways (atelectrauma). To investigate the relationship of the pathological behavior of ALI/ARDS with that of various mechanical ventilation regimes, a reduced-dimension multi-scale model was developed for an anatomically based airway tree that incorporates realistic tube-law behavior and interfacial phenomena to identify airway recruitment/derecruitment processes that can influence VALI.

Materials and Methods: The domain for the lung model was generated based on morphological data from Lambert *et al.* (1982) and the airflow in the airways were assumed to have flow behavior driven by a sinusoidally varying pleural pressure. A hydraulic circuit analogy was implemented with resistance due to viscous flow and liquid obstruction. The airway geometry was governed by a tube law developed by Lambert *et al.* (1982) that described the cross sectional area as a function of the transpulmonary pressure. To characterize the lung and respiratory system pressure-volume (P-V) relationship, a sigmoidal equation was used as described by Venegas *et al.* (1998) and Fujioka *et al.* (2013). This allowed for P-V curves that described a variety of conditions including normal respiration and mechanically ventilated patients with acute respiratory distress syndrome. The model also includes mechanisms for airway closure with the formation and movement of liquid plugs in the more compliant regions of the airway tree, as well as reopening of airways following the rupture of a liquid plug. Additionally, Langmuir kinetics were utilized to prescribe the surface tension of each airway based on the surfactant concentration within the interfacial film.

Results and Discussion: We simulate normal respiration and ventilation under ALI/ARDS conditions, which yield time and spatially dependent closure and reopening events. In particular, the effects of the sensitivity of the pressure distribution combined with that of variable surface tension throughout the tree on airway closure are examined. Additionally, the effects of liquid plug movement and rupture on ventilation behavior were observed and elucidate associated timescales of obstruction clearance in the lung.

Conclusions: The information obtained from this model yields knowledge of primary parameters affecting overall lung stability and illuminate the mechanisms necessary to alleviate damage during mechanical ventilation. Further work will implement more complex flow regimes as well as parenchymal tethering and fluid-structure interactions at the airway wall. This will lead to the ability to more accurately simulate the effects of atelectrauma and barotrauma on the lung.

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On Three Sources of Inaccuracy in Large-Scale Molecular Mechanics Simulations

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This paper presents a particular experimental investigation of the computer simulations behavior in classical molecular dynamics and quantum molecular mechanics. These methodologies are modern tools in major research areas like physics, chemistry and biology. The focus of this study is entirely and solely on the issues posed by the limited nature of computation to the mathematical and algorithmic requirements of the methods. Three factors: computation precision, integration step, and boundary conditions and their respective influence on the deviation of simulation results are considered for extremely long running simulations.

The obtained high levels of deviation show how carefully and with compromise to performance and resources should the values for those parameters be chosen, especially in the long running cases.

All experiments are conducted using only widely adopted open source software packages, which allows combining the confidence in the well established methodologies with their implementation and the ability to make the necessary adaptation, in order to achieve the goals of the testing.

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Integrable Equations on Symmetric Spaces, Generalized Fourier Transforms and Mikhailov Reductions

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We start with a Riemann-Hilbert problem (RHP) related to a **BD.I**-type symmetric spaces $SO(2r+1)/S(O(2r-2s+1)\otimes O(2s))$; for simplicity we put $r=4$, $s=3$. Next we consider the Lax operator:

$$L\psi \equiv \left(i\frac{\partial}{\partial x} + \begin{pmatrix} 0 & \boldsymbol{q} & 0 \\ \boldsymbol{q}^\dagger & 0 & \tilde{\boldsymbol{q}} \\ 0 & \tilde{\boldsymbol{q}}^\dagger & 0 \end{pmatrix} - \lambda \begin{pmatrix} \mathbb{1} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\mathbb{1} \end{pmatrix} \right) \psi(x,t,\lambda) = 0,$$

Next we apply $\mathbb{Z}_6$ Mikhailov reduction, after which $Q(x,t)$ depends on only two complex functions q_1 and q_2. This results in a new 2-component NLS type system with Hamiltonian [1]:

$$H = \left|\frac{\partial v_1}{\partial x}\right|^2 + \left|\frac{\partial v_2}{\partial x}\right|^2 - \frac{1}{2}\left(|v_1|^2 + |v_2|^2\right)^2 + \frac{1}{2}\left(v_1 v_2^* - v_1^* v_2\right)^2,$$

where $v_1 = \sqrt{6}q_1$, $v_2 = \sqrt{3}q_2$.

The spectral properties of the reduced Lax operator L and the fundamental properties of the relevant class of nonlinear evolution equations are described. We prove that the mapping from the potential $Q(x,t)$ to the scattering data of L may be viewed as a generalized Fourier transform. Using them one is able to derive all fundamental properties of these equations. These include the description of the class of NLEE through the recursion operators and their bi-Hamiltonian properties of the relevant NLEE. Other examples of new 2-component NLS equations related to other types of symmetric spaces are presented in [2].

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High-Performance Computing on GPUs for Resistivity Logging of Oil and Gas Wells

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At present, the development of geophysical well logging techniques requires the use of highly effective programs for mathematical simulation. The utilization of algorithms for solving computational logging problems in realistic mathematical statements makes it possible to significantly improve the accuracy of interpretation results when studying complex geological objects. However, their widespread implementation into practice is complicated, due to the high resource intensity and poor performance. Processing and interpretation of practical data in real time is possible on the basis of high-performance computations on graphics processors (GPUs).

The paper deals with the widely used in Russia electrical lateral logging sounding method (BKZ) designed to study the electrical resistivity of the rocks. The BKZ method is based on the principle of radial sounding of a geological medium with subsequent assessment of the fluid saturation of rocks is ultimately performed.

We have obtained numerical solutions of the direct BKZ problem, based on the finite difference and finite element methods. The numerical solutions of the original Poisson equation with the help of the grid methods are reduced to systems of linear algebraic equations (SLAE) with sparse matrices of large dimension. To solve them, we apply a direct method of the Cholesky decomposition. The features of implementation of the algorithms for GPU calculations are investigated, applying NVIDIA CUDA. For instance, the application of the direct method allows to decompose the matrix once, and then to use it for finding several solutions of the SLAE.

Evaluations of CPU and GPU performance, including heterogeneous CPU-GPU computations, are performed. It is established that with the use of heterogeneous CPU-GPU calculations one can improve the performance compared to similar calculations on a CPU or GPU. The results of the performed studies indicate the high efficiency of the developed algorithms for solving a wide range of practical resistivity logging problems.

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ArctosPop: Programme for Estimating the Brown Bear (Ursusarctos) Population Size in Bulgaria

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Monitoring populations of protected wildlife species is necessary for effective management and conservation of their habitats. One of the best habitats of brown bears (Ursusarctos) which are a strict protected species in Europe is located in Bulgaria.

In this work we present the program tool ArctosPop for automatic estimation of the brown bear (Ursusarctos) population size in Bulgaria. This computing programme integrates statistical algorithms which use as input data the observed data for traces of brown bears during National monitorings. As future work the guidelines for improvement of this programme tool are also presented.

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The Interaction of a Vortex Pair with A Free Surface

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Here we study the interaction of vortex pair with a free surface. The theoretical and laboratory researches are performed [1]. With the creation of supercomputers, it became possible to investigate such problems more accurate. Bearing in mind, in such flows, there are areas with large gradients of hydrodynamic parameters, the required methods should possess such properties as a high order of accuracy, a minimum scheme dissipation and dispersion, as well as monotonicity. This paper will provide parametric calculations of interaction of the vortex pair with a free surface in case of homogeneous fluid with different distributions of vorticity. Calculations will be held with SMIF method (Splitting on physical factors Method for Incompressible Fluid flows) [2] that possesses the properties mentioned above. Eventually the calculations will be compared with some theoretical, experimental data and calculations of other authors. As a result researching of spatial structures on free-surface forming by emerging vortexes in homogeneous fluids it could be used in environmental monitoring and forecasting preventing and eliminating pollution, as well as distributed software technology and high performance computing and to address a number of practical problems.

Acknowledgement. This work has been partly supported by Russian Science Foundation (grant No. 17-11-01286).

Keywords: Mathematical modeling, vortex structures, free-surface, parallel computing.

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Non-Classical Lie Symmetry Analysis to the Nonlinear Time-Fractional Differential Equations

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This study is devoted to a class of linear and nonlinear differential equations with time fractional order. By employing classical and non-classical Lie symmetry analysis and some technical calculations; new infinitesimal generators are obtained which gives rise to derive the invariant solutions for this class of equations.

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The Method of Trend Analysis of Gas Turbine Engine Parameters in Long-Term Operation

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For analyzing the time series of the measured parameters of gas turbine engine in long-term operation, the method is proposed, which differs from known methods by using the engine throttle performances. The time series of the speeds of engine's high-pressure turbine and engine's low-pressure turbine, inlet and outlet pressures, outlet gas temperatures, fuel consumption, vibration parameters in each flight cycle at take-off, cruising and half-power modes are the initial data for the analysis.

The time unevenness of the counts is a distinctive feature of the time series of aircraft engines parameters, since the flight cycles are separated by unequal time intervals (from several hours to several days). Therefore, the initial time series are transformed into arrays of the measured parameters deviations from the calculated parameters obtained from the throttle performances of a particular engine. Such throttle performances are given in the technical documentation for the engine and obtained at its development testing. The deviation arrays, ordered by flight cycles, are analyzed by known methods (SSA, caterpillar, main component, *etc.*) for selecting the components with the maximum variances. Such components characterize the throttle performances changes (deformations) during engine operation. An interval estimations of the trend components at a given level of statistical significance were performed. The known trend criteria (Abbe, cumulative sums and others) are used for determining the trend presence. Based on the processing results a diagnostic message about the trend absence is generated with a given confidence probability.

The solution of the applied problem of analyzing the operating conditions of a specific aircraft gas turbine engine is considered. The proposed method makes it possible to increase the reliability of statistical conclusions about the operating conditions of gas turbine engines in long-term operations.

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Biomolecule Modeling: Conformation Studies in Coarse-Grain Approaches

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In biomolecule modeling, computational challenges are based on the system size on the one side and the characteristic time scales on the other. The necessity for time steps of the order of a few femtoseconds and the nature of the underlying biophysical and biochemical processes poses severe limitations on the time span of the investigated phenomena. We discuss the possibilities for efficient acceleration of biomolecule simulations by appropriate system representations. In particular, we analyze the applicability of coarse-graining approaches in the investigations of conformation-dependent processes, to put forward the advantages of synergistic protocols from complementary approaches that are able to extend the boundaries of the underlying physical modeling techniques. We exemplify our considerations by two cases: a study of tagging influence on human interferon gamma binding affinity within MARTINI coarse-grain approach and protein folding simulations of the villin headpiece by a single-stage extended MD run and within a mutliscale MC-MD soliton-based approach.

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A Dressing Method for The Camassa-Holm Equation

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We present the derivation of the soliton and cuspon solutions of the Camassa-Holm equation [1] by the implementation of the dressing method [2]. The one and two soliton/cuspon solutions as well as soliton-cuspon interactions will be discussed.

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Inverse Log-Gamma-G Processes

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The paper starts with definition and investigation of univariate and multivariate Inverse Log-Gamma processes. Some new properties of time intersections are obtained. The influence of the parameters is visualized. The finite dimensional distributions and increments are described. Following the ideas about G-classes of distributions and making transformations of random processes, we define Inverse Log-Gamma-G processes. As particular cases, Inverse-log-Gamma-G processes with G: Exponential,

Frechet, Weibull, Gumbel, Pareto are considered. The relation with extreme value theory is described.

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Semi-Analytical Investigation of Unsteady Free-Boundary Flows

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The methods which are intermediate between purely analytical and purely computational ones belong to a semi-analytical group, according to the classification by Milton Van Dyke. They are mainly the methods of processing the series and sequences. This group comprises the well-known algorithms based on the Padé approximants and continuous fractions as well as convergence acceleration algorithms, Domb-Sykes test, *etc.* The authors use this technique to investigate unsteady inviscid incompressible free-boundary flows. The solution is obtained in the form of a power series in time, the coefficients for it being computed. The obtained series are summarized and singularities are revealed.

A general problem which may be solved using power series in time consists in finding deformations of the domain occupied by a liquid if pressure of the free boundary is constant (surface tension is absent). An initial configuration of the domain and an initial velocity field are assumed to be known. Conformal mapping of the fixed domain onto the flow region is sought in the form of power series or the Laurent series. About a thousand of terms of a series with length of mantissa 1300 decimals are found. The change of variables of special form with subsequent Padé summation is used. The free boundary, velocity characteristics and other parameters of the flow at each moment of time are found.

The greatest attention devoted to the problems of deformation of cavities in infinite liquid. Let for time $t = 0$ there be a circular cylindrical cavity of unit radius in a liquid which moves with a unit velocity (liquid at infinity is at rest). It is necessary to find a deformation of the cavity when $t > 0$ if zero pressure is maintained inside it and the cavity volume is constant.

The proposed model seems to be very promising. Although complicated in terms of application, it admits a deep research of the problem. It allows a free surface to be constructed for large times and the appearance of singularities on it to be recorded.

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An Iterative Solution of the Inverse Cauchy Problem for Elliptic Equations by Conjugate Gradient Method

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The article presents the results of computational experiment carried out using a finite-difference method for solving the inverse Cauchy problem for a two-dimensional elliptic equation. The computational algorithm involves an iterative determination of the missing boundary condition from the overdetermined condition using the conjugate gradient method. The results of calculations carried out on the examples with exact solutions as well as at specifying an additional condition with random errors are presented. They showed a high efficiency of the iterative method of conjugate gradients for numerical solution of the set problem

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Modeling the Effects of the Immune System on Bone Fracture Healing

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A mathematical model is presented for the bone fracture healing process. The model incorporates the mechanisms and functions of the immune-system cells at the fracture site. The resulting system of nonlinear ordinary differential equations is studied analytically and numerically. Mathematical conditions for a successful bone repair are formulated. The model is also used to simulate the progression of the healing process and to examine the effect of inflammatory cytokines in bone fracture repair.

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The Performance Rating of OpenMP and Corresponding MPI-2 Routines on The Cluster Platform "Avitohol"

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In this paper it will be consider the performance and scalability of the OpenMP and corresponding MPI-2 routines using a high-performance computing system "Avitohol." It will be used two different benchmark tests which represent commonly used communication patters. The range of the patterns will be from light to heavy communication traffic like: circular shift and distant messages passing. For this experiment, the Avitohol system will be used, consisted of 150 HP Cluster Platform SL250S GEN8 servers with 2 Intel Xeon E 2650 v2 CPUs and 2 Intel Xeon Phi 7120P co-processors. The synchronization mechanisms are necessary when using one-side communication and overhead an implementation. Each implementation will be run using message sizes of 16 bytes, 32 Kbytes and 1Mbyte. The results of the tests that we'll run on the Avitohol will represent the difference in the performance achieved with OpenMP and MPI-2 implementations. It will be reviewed the performance and scalability of previous versions of the MPI and OpenMP and will compare the results with this ones, received from this experiment. This information can be very helpful in cases we need to implement specific applications on a Linux cluster and the best implementation method needs to be defined.

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Orbital Stability or Instability of Solitary Waves to Boussinesq Paradigm Equation with Quadratic-Cubic Nonlinearity

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In this talk the solitary waves to Boussinesq paradigm equation with quadratic-cubic nonlinearity

$$u_{tt} - u_{xx} - h_2 u_{ttxx} + h_1 u_{xxxx} + u + (au^2 + bu^3)_{xx} = 0 \qquad (1)$$

are given.

The orbital stability or instability of solitary waves with velocities c in case $h_2 > h_1$ is completely studied by the Grillakis-Shatah-Strauss stability theory. Explicit formulas for functions $d(c)$ and $d''(c)$, connected with the conserved energy and the momentum of equation (1), are derived. By means of these formulas, orbital stability or instability of solitary waves is proved.

Acknowledgement. This research is partially supported by the Bulgarian Science Fund under grant DFNI I-02/9.

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Convective Instability of Non-Isothermal Couette Flow Between Two Rotating Cylinders Caused by Internal Heat Generation in Accordance with the Arrhenius Law

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Consider a vertical annulus between two concentric cylinders filled with a viscous incompressible fluid. The inner cylinder is rotating with constant angular velocity while the outer cylinder is at rest. Heat sources are distributed within the fluid in accordance with the Arrhenius law. The annulus is closed (the fluid flux through the cross-section of the channel is zero).

There exists a steady convective motion in the vertical direction caused by rotation and heat generation. Two velocity components (in the vertical and azimuthal directions) and temperature depend on the radial coordinate only. The steady state is described by a nonlinear boundary value problem which is solved numerically.

Stability of the steady convective motion with respect to small perturbations is investigated. The method of normal modes is used to derive the system of ordinary differential equations from the linearized equations for small perturbations. The corresponding boundary value problem is solved numerically using collocation method. Marginal stability curves are obtained for different values of the problem. Different instability regimes in the parameter space are identified. It is shown, in particular, that the increase in the Prandtl number and Frank-Kamenetsky parameter destabilizes the flow.

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On Feedback Target Control for Uncertain Discrete-Time Bilinear Systems with State Constraints through Polyhedral Techniques

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We deal with problems of terminal target control synthesis for discrete-time systems under uncertainties and state constraints. We use the deterministic model of uncertainty with set-membership description of the uncertain items when there is no any statistical information. The set-membership models of uncertainty may be used in many applied areas. It is known that the main items in treating problems of the mentioned kind are so called solvability tubes, which are set-valued functions. Since practical construction of such tubes may be cumbersome, different numerical methods were devised. Among them constructive computation schemes for linear systems based on the ellipsoidal techniques were proposed by A.B. Kurzhanski and then expanded to the polyhedral techniques by the author. Their main advantage is that such techniques allow to find solutions by rather simple means.

Here we develop the methods of control synthesis for discrete-time bilinear systems using polyhedral solvability tubes. We consider the systems in which the controls appear additively in the right hand sides of the system equations and in the coefficients of the system. The controls are subjected to given constraints of parallelepiped-valued and interval types respectively. We assume that there are uncertain terms of two similar types in the system. Moreover the systems are considered under state constraints, which are described in terms of zones (*i.e.*, intersections of strips). Note that the systems with controls (or/and uncertainties) in the system matrix are of bilinear type and have properties of nonlinear systems (in particular reachable sets and solvability sets of such systems can be non-convex). We present the recurrence relations which describe the polyhedral (parallelotope-valued) solvability tubes. Control strategies, which can be constructed using the mentioned tubes, are proposed. Also we provide some illustrative examples.

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Necessary and Sufficient Conditions for Finite Time Blow Up of The Solutions to Double Dispersion Equation

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Necessary and sufficient conditions for finite time blow up or global existence of the solutions to the Cauchy problem for the double dispersion equation are proved. Combined power type nonlinearities, including the quadratic-cubic nonlinearitiy, are considered. Easy checkable sufficient conditions for finite time blow up of the solutions are derived. Additionally, initial data with arbitrary positive energy, which satisfy the sufficient conditions are explicitly constructed.

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Numerical Study on the Turbulent Mixing Due to RT and RM Instabilities in The Imploding Problem

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In this paper, the turbulent mixing due to the Rayleigh-Taylor (RT) and Richtmyer-Meshkov (RM) instabilities have been studied by using high-resolution Euler numerical methods. Simulations of two imploding models have been performed. One model is driven by a shock wave, and both the RM and RT instabilities occur during the imploding and mixing process. The other is a shell's quasi-isentropic compression problem, for which the turbulent mixing is induced only by the RT instability. We have discussed the effects of the initial perturbation, premixing and distortion on the mixing's growth rate and the 'atom mixing' degree. We found that for both cases there is no apparent approach to a self-similar regime independent of the initial perturbation spectrum. The width of the turbulent mixing zone (TMZ) is quite sensitive to the initial perturbation scale, and it grows more slowly with smaller scale perturbations. The TMZ's 'atom mixing' degree is also sensitive to the initial perturbation scale, but the difference due to initial perturbation tends to decrease with time. The premixing will reduce the mixing's growth rate during the deceleration regime and the explosion regime, and it will enhance the TMZ's 'atom mixing'. Because of the evolution of the low-order mode perturbations, more local mixing and ejecta will emerge for the distorted shell as compared with the one with no initial distortion.

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Studying The Asymmetry in the hIFNγ Homodimer Using Molecular Dynamics Simulations

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Interferon gamma (IFNγ) is an important cytokine, which plays a key role in the formation and modulation of immune response. Human IFNγ ((hIFNγ) is a 17kDa single polypeptide protein consisting of 143 amino acids (aa), organized in six α-helices (comprising 62% of the molecule), which are linked by short unstructured regions. Besides them, hIFNγ contains also a long positively charged unstructured C-terminal domain composed of 21 aa.

Under physiologic conditions the mature form of hIFNγ is organized as a non-covalent homodimer, in which the two subunits are associated in an antiparallel orientation. The cytokine accomplishes its effects via high-affinity extracellular interaction with its specific receptor (IFNγR1). It has been shown that the hIFNγ dimer interacts with two IFNγR1 receptor molecules, forming a symmetric $1:2$ active complex.

We studied the conformation of hIFNγ using by molecular dynamics (MD) simulations. The trajectories were analyzed using the Spatiotemporal Multistage Consensus Clustering (SMCC) method. SMCC is a post processing technique for analysis of MD trajectory data, that identifies compact groups of amino acid residues forming semi-rigid domains. The SMCC method consists of initial spatial clustering performed on successive subsegments of an MD simulation trajectory, followed by a temporal self-consistent consolidation of the results to generate domains that are stable over time.

The SMCC analysis identified different semi-rigid domains in the two monomers of the hIFNγ dimer. They exhibits neither chiral nor mirror symmetries. The conformations of two monomers in the initial structure are almost identical with and RMSD of less than 1Å. Yet they evolve differently as demonstrated by the cluster analysis results. Understanding of the causes of this asymmetry necessitates further simulations and analysis.

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Generalized Mathematical Model of Heat Conduction in a Complex Multi-Layered Area

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Determination of the temperature field in multilayer area with different thermophysical characteristics leads to the solution of boundary value problems with boundary conjugation conditions. Particular difficulties arise during the solution of such problems in non-canonical area. The mathematical model of the temperature field in a multilayer cylinder, which describes the process of heat transfer in an electric machine, is considered. It is assumed that the temperature field of cylinder in the mathematical model does not depend on the axial coordinate. On the odd layers of the cylinder, internal heat sources act, from which heat is transferred to even layers by thermal conductivity. The geometric area, the multilayer cylinder, is intersected by planes that are perpendicular to the axis of the cylinder. The temperature field along the radius of each section of the cylinder is investigated. This leads to the solution of the set of initial-boundary value problems for the heat equation with boundary conditions of the impedance type, which are simultaneously conjugate conditions. Further reduction of the solutions around the cylinder is carried out.

The constructed mathematical model is presented in the form boundary value problem for the nonstationary heat equation in a four-layer cylinder with initial distribution, and nonlinear boundary conditions at the outer boundaries.

Internal heat sources on the first and third layer are represented in the form of Joule heat, which is released in the windings of the electric machine. The solution of the problem is obtained by a numerical-analytical method with the construction of implicit difference schemes. Based on the results of numerical experiments, graphs of temperature distributions along the sections of the cylinder (electric machine) were constructed. The temperature field of an electric machine is constructed by the reduction method.

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Treating Outliers in SARIMA Models to Forecast Dengue Cases for Age Groups Ineligible for Vaccination in Baguio City

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Dengue infection remains to be one of the most significant health issues in the country. Despite the availability of vaccine since 2016, individuals less than 9 years old and adults greater than 45 years old remain susceptible to the mosquito-borne disease. An accurate forecast in the future dengue cases from susceptible age groups would greatly help in the efforts to prevent further increase in infections. Outliers were observed from the data and were treated using different techniques which were used to create a SARIMA model. Initial findings suggests that the best model is created using winsorization with logarithmic transformation of the data and the corresponding SARIMA model is ARIMA(1,1,0)(2,0,0)$_{12}$. This model is then used to forecast the monthly dengue cases for Baguio City for a year for the age group considered.

Keywords: Outliers, univariate, time Series, dengue, ARIMA

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On one Algorithm for Solution to the Control Problem Using Sets of Guides

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This work considers the dynamical system and the problem of finding the initial position for this system in n-dimensional Euclidean space for which it is possible to reach the target set on the defined finite time interval. Furthermore, it is formulated the problem of constructing the control which brings the system from the initial position to the target set. The problem of finding the initial position and the corresponding control is a topical in control theory and useful in such fields as robotics, economics, ecology, *etc.* At this time the method of solving this problem based on discrete representation of time and space is proposed. In this method the time interval is substituted with some partition and the system is being considered only at the moments from this partition. Furthermore the space is divided with the constant grid and the sets are represented as the sets of nodes of this grid. When the size of grid and the step of the time partition are small the time of computation is often excessive. It is following from the necessity to process huge amounts of data points. Especially this problem is acute for the dimension of space higher than 4. In order to reduce the computation time the 2-stage step-by-step method is proposed allows to construct the piecewise-constant control which brings the system to the defined neighborhood of the target set. On the first stage of this method the resolvability sets are approximately constructed for each time moment from the introduced time partition. During this process the rectangular space grid with the fixed amount of cells is used. The size of this grid coincide with the size of the set itself and the size of the cell depends on the geometry of the set. This approach allows to reduce amount of points participating in the calculation process which in turn reduces the time of computation. On the second stage of the method some arbitrary point from the resolvability set calculated for the initial time moment is chosen as the initial position. Finally, the piecewise-constant control solving the problem of reaching the target set with the predefined accuracy is constructed using the points from the resolvability sets as the sets of guides.

Acknowledgement. This research was supported by Russian Science Foundation (Projects 15-11-10018)

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Ellipsoidal Estimates of Reachable Sets of Impulsive Control Problems under Uncertainty

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The problem of ellipsoidal estimating reachable sets of impulsive control systems with bilinear nonlinearity and with uncertainty in initial states is studied. It is assumed that we know the bounding set for initial system states and any additional statistical information is not available. Also the matrix included in the differential equations of the system dynamics is uncertain and only bounds on admissible values of this matrix coefficients are known. Under such conditions the dynamical system is nonlinear and reachable set loses convexity property. Such systems may be found in many applied areas such as engineering problems in physics and economics, biological and ecological modeling when it occurs that a stochastic nature of the errors is questionable. For instance, in case of limited data or after some nonlinear transformation of the data, the presumed stochastic characterization is not always valid. Hence, as an alternative to a stochastic characterization a so-called set-membership approach, has been proposed and intensively developed in the last decades. Here we develop the set-membership approach based on ellipsoidal calculus. This approach allows to find the effective estimates of reachable sets for models with linear dynamics under presented set-membership uncertainty. Using results of the theory of trajectory tubes of control systems and techniques of differential inclusions theory and using the special structure of uncertainties, we present here approaches that allow finding set-valued estimates of related reachable sets of considering impulsive uncertain control system. The algorithms of constructing the ellipsoidal estimates for studied systems are given. Numerical simulation results related to the proposed techniques and to the presented algorithms are given.

Acknowledgement. The research was supported by Russian Science Foundation (RSF) (project No.16-11-10146).

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Recent Splitting Schemes for The Incompressible Navier-Stokes Equations

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The talk will be focussed on two classes of recently developed splitting schemes for the Navier-Stokes equations.

The first class is based on the classical artificial compressibility (AC) method. The original method proposed by J. Shen in 1995 reduces the solution of the incompressible Navier-Stokes equations to a set of two or three parabolic problems in 2D and 3D correspondingly. Unfortunately, its accuracy is limited to first order in time and can be extended further only if the resulting scheme involves an elliptic problem for the velocity vector. Recently, together with J.L. Guermond (Texas A&M University) we proposed a scheme that extends the AC method to any order in time using a bootstrapping approach to the incompressibility constraint that essentially requires to solve only a set of parabolic equations for the velocity. The conditioning of the corresponding linear systems is therefore much better than the one resulting from an elliptic problem for the velocity.

The second class of methods is based on a novel approach to the Navier-Stokes equations that reformulates them in terms of stress variables. It was developed in a recent paper together with P. Vabishchevich (Russian Academy of Sciences). The main advantage of such an approach becomes clear when it is applied to fluid-structure interaction problems since in such case the problems for the fluid and the structure, both written in terms of stress variables, become very similar. Although at first glance the resulting tensorial problem is more difficult, if it is combined with a proper splitting, it yields locally one dimensional schemes with attractive properties, that are very competitive to the the most widely used schemes for the formulation in primitive variables. Several schemes for discretization of this formulation will be presented together with their stability analysis.

Finally, numerical results for a problem with a manufactured solution will be presented.

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Results on Applications of Neutrix Calculus to Some Operations on Distributions

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We give a meaning to the symbols such as $\delta^k(x)$, $[\delta^{(r)}(x)]^{-k}$, $[\delta_+^{(r)}(x)]^{-k}$, $\delta^{(r)}(H(x))$, $H^\lambda(x)$ $(\lambda \in \mathbb{R})$ and $\delta^k(f(x))$, where $f(x)$ is an infinitely differentiable function.

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Comparative Analysis on The Probability of Being a Good Payer

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Credit risk assessment is crucial for the bank industry. The current practice uses various approaches for the calculation of credit risk. The core of these approaches is the use of multiple regression models, applied in order to assess the risk associated with the approval of people applying for certain products (loans, credit cards, *etc.*). Based on data from the past, these models try to predict what will happen in the future. Different data requires different type of models. This work studies

the causal link between the conduct of an applicant upon payment of the loan and the data that he completed at the time of application. A database of 100 borrowers from a commercial bank is used for the purposes of the study. The available data includes information from the time of application and credit history while paying off the loan. Customers are divided into two groups, based on the credit history: “Good” and “Bad” payers. Linear and logistic regression are applied in parallel to the data in order to estimate the probability of being good for new borrowers. A variable, which contains value of 1 for “Good” borrowers and value of 0 for “Bad” candidates, is modeled as a dependent variable. To decide which of the variables listed in the database should be used in the modeling process (as independent variables), a correlation analysis is made. Due to the results of it, several combinations of independent variables are tested as initial models – both with linear and logistic regression. The best linear and logistic models are obtained after initial transformation of the data and following a set of standard and robust statistical criteria. A comparative analysis between the two final models is made and scorecards are obtained from both models to assess new customers at the time of application.

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Stability of Linear Multistep Methods

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We investigate the stability of linear multistep methods from a new direction. The usual approach is to reformulate the method to a multidimensional one-step method and investigate its companion matrix. Our approach is different since we handle the whole process as one matrix equation similarly to the FDM approximation of boundary value problems. First, we demonstrate the usefulness of our approach by applying it to fixed stepsize linear multistep methods. Second, we use our approach to handle variable stepsize linear multistep methods and we show how to obtain this way stepsize restrictions which ensure the stability.

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On the Numerical Solution *vs.* Discrete-Event Simulation of Fluid Stochastic Petri Net Models

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The concept of fluid models was used in the context of Stochastic Petri Nets, referred to as Fluid Stochastic Petri Nets (FSPNs), where variables are represented by fluid places, which can hold fluid rather than discrete tokens. Transition firings are determined by both discrete and fluid places, and fluid flow is permitted through the enabled timed transitions in the Petri Net. By associating exponentially distributed or zero firing time with transitions, the differential equations for the underlying stochastic process can be derived. The dynamics of an FSPN are usually described by a system of first-order hyperbolic partial differential equations (PDEs) combined with initial and boundary equations. The general system of PDEs may then be solved by a standard discretization approach, while the transportation of fluid in zero time is described by appropriately chosen boundary conditions.

An FSPN-based analytical model of an Instruction Level Parallel (ILP) processor with aggressive use of prediction techniques and speculative execution is taken as an example, for which the state equations for the underlying stochastic process are derived, and performance evaluation results are presented. Namely, considering a machine that employs multiple execution units capable to execute large number of instructions in parallel, the service and storage requirements of each individual instruction are small compared to the total volume of the instruction stream. Individual instructions may then be regarded as atoms of a fluid flowing through the pipeline. Both the application of finite-difference approximations for the partial derivatives, as well as the discrete-event simulation of the proposed FSPN model, allow for the evaluation of a number of performance measures. The numerical solution makes possible the probabilistic analysis of the dynamic behavior, although the advantage of the discrete-event simulation is the much faster generation of performance evaluation results.

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Econometric Modeling of The Impact Of Religion on Macroeconomic Parameters

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This article presents the result of an econometric model of the impact of religion and religious beliefs on macroeconomic parameters. The ongoing religious imbalance in the development of society, along with the growing spread of individual denominations, caused by increased migration flows and increased antagonism and rejection affect micro and macroeconomic parameters. Thus, according to the scientists of the American research center Pew Research Center, by 2050 one of the main risks of economic development will be the change in the confessional structure of society. According to their forecast, the total number of Muslims will increase by 73%, and Christians only by 35%. This circumstance makes it urgent and necessary to conduct empirical studies to determine the role of religious beliefs and interfaith differences in comparison with other socioeconomic processes and to identify their impact on micro and macroeconomic indicators.

We developed models of the influence of religion on basic economic indicators: GDP and the index of human development (HDI). All countries in three groups: a general sample, countries with a population professing Islam, a country with a population professing Christianity. The key factor chosen in the study was religion. Religion was calculated as the percentage of people recognizing the dominant importance of religion in their lives.

As a basic model, we considered the model of Valeriani and Peluso (2011)). As fundamental units for the GDP model, we took final consumption (C), gross capital formation (I), public expenditure (G) and net exports (Xn) for GDP by the method of calculating expenditures. For the HDI as aggregators, we investigated the costs of science (RD) and life expectancy. The econometric models were tested by us for the Hausman test and demonstrated a determination coefficient from 0.7 to 0.93, which is sufficient for econometric studies. For the GDP model in the general sample of Christian countries, religion was insignificant, and in Muslim countries, religion is among the most significant variables. At the same time, religion positively influenced the GDP of Islamic states For the HDI model, in a general sample of Christian and Muslim countries, religion was a significant variable and influenced the dependent variable positively. At the same time, for Christian countries, religion, turned out to also be a significant variable, which affected the HDI negatively.

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The Impact of Maki Parameter and Spin Orbit Scattering Constants on The WHH Model of Upper Critical Magnetic Fields in Ni- and Co- doped Pnictide Bulk Superconductors

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Werthamer, Helfand, and Hohenberg (WHH) model includes the effects of both Pauli paramagnetism and spin-orbital scattering to predict the universal behavior of the upper critical field $Hc_2(T)$ in superconductors with weak electron-phonon coupling. This approach incorporates both orbital and paramagnetic effects in the upper critical field's temperature dependence by approximately evaluating the non-local non-magnetic and spin-orbit scattering integrals that enter the self-consistency equation for the gap function. The relative importance of the orbital and paramagnetic effects in the suppression of the superconductivity is described by the Maki parameter $I_{\pm}$. In most superconductors, the Maki parameter is usually much less than unity and this indicates that the influence of the paramagnetic effect is negligibly small. However, in materials with heavy electron effective mass, in which the Fermi energy is small, or in layered materials in a magnetic field parallel to the layers, $I_{\pm}$ can be larger than unity. This is the case for our bulk pnictides. By fitting the measured temperature dependence of the upper critical field, we derive the Maki parameter and the spin orbit scattering constant and assess how these vary with Co and Ni doping, as we search for clues as to possible future enhancements of these materials. We examine the correlation between the Maki parameter, the spin orbit scattering constant, and the reduced upper critical field for a variety of curve fits.

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Evolution of Nonlinear Waves in a Blood-filled Artery with an Aneurysm

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We discuss propagation of traveling waves in a blood- filled elastic artery with an axially symmetric dilatation (an idealized aneurysm). The processes in the injured artery are modelled by equations for the motion of the wall of the artery and by equation for the motion of the fluid (the blood). Taking into account the specific arterial geometry the model equations are reduced to a version of the perturbed Korteweg-deVries equation with variable coefficients. Exact travelling-wave solutions of this equation are obtained by the modified method of simplest equation where the differential equation of Abel is used as a simplest equation. A particular case of the obtained exact solution is numerically simulated and discussed from the point of view of arterial disease mechanics.

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Asymptotic Estimation of Free Vibrations of Nonlinear Plates with Complicated Boundary Conditions

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We consider the modified method of parameter continuation (MMPC) as an asymptotic technique for estimating the frequencies and shapes of nonlinear oscillations of plates with complicated boundary conditions. Unlike the Bolotin method, which is usually used for such estimations, MMPC estimations depends from the shape of initial perturbation. When the perturbation coincides with the eigenform of plate vibration then its frequency is close to the relevant eigenfrequency. In another cases, however, it describes real vibration shape of plate with different conditions on its edges. For many important cases the vibration frequency is not

equal for all points of plate. The comparison with numerical calculations done on Solid Works package confirms the advantages of proposed method and its accuracy.

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Study of Eigenfrequencies with The Help of Prony's Method

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Eigenfrequencies can be crucial in the design of a construction. They define many parameters that determine limit parameters of the structure. Exceeding these values can lead to the structural failure of an object. It is especially important in the design of structures which support heavy equipment or are subjected to the forces of airflow. One of the most effective ways to acquire the frequencies' values is a computer-based numerical simulation. The existing methods do not allow to acquire the whole range of needed parameters. It is well known that Prony's method, is highly effective for the investigation of dynamic processes. Thus, it is rational to adapt the Prony method for such investigation. The Prony method has advantage in comparison with other numerical schemes because it provides the possibility to process not only the results of numerical simulation, but also real experimental data. The research was carried out for a computer model of a steel plate. The input data was obtained by using the Dassault Systemes SolidWorks computer package with the Simulation add-on. We investigated the acquired input data with the help of the Prony method. The result of the numerical experiment shows that Prony's method can be used to investigate the mechanical eigenfrequencies with good accuracy. The output of Prony's method not only contains the information about values of frequencies themselves, but also contains data regarding the amplitudes, initial phases and decaying factors of any given mode of oscillation, which can also be used in engineering.

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Fuzzy Recognition Method of High Molecular Substances in Evidence-Based Biology

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Modern requirements to reliability achievement and high quality of researches put mathematical analysis methods of results at the forefront. In connection with this, evidence-based methods of processing experimental data have become increasingly popular in the biological sciences and medicine. Their basis is meta-analysis, a method of quantitative generalization of a large number of randomized trails on a specific problem, which are often contradictory and performed by different authors. It allows identifying the most important trends and quantitative indicators of the data. Meta-analysis uses heterogeneous research, which makes it possible to verify the advanced hypotheses and to discover new effects in the population genotype. The existing methods for recognizing high molecular substances by gel electrophoresis of proteins under denaturing conditions are based on approximate methods for comparing the contrast of electrophoregrams with a standard solution of known substances. To increase the accuracy and validity of the findings on the detection of new proteins, we propose a fuzzy method for modeling experimental data.

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Solving the Displacement problem in a Geodesic Polygon with Exact Length Edges and Small Angle Errors

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A geodesic polygon is a sequence of points (stations) and vectors, defined by consecutive geodesic measurements between each two consecutive points. The polygon is closed if its starting and ending points are the same, otherwise it is open. We consider the solution of displacement problem in open and closed azimuth polygons, in which some errors occur in measuring the vector directions, while their lengths are exact. Such problems appear in practice in placing long polygonal constructions, made of rigid linear elements of fixed (chosen in advance) lengths, in tight closed spaces of natural or artificial objects. Possible applications include, for example, placing a water pipe or other rigid communication in a tunnel, construction of touristic trial (consisting of ladders and bridges) throughout a canyon or a cave, *etc.* On the first stage, when the project of the construction should be done and the elements must be ordered afterwards, geodesic survey of the object is performed. Due to certain specifics of the object, exact measurement of the azimuths is either too expensive or even impossible to be done on this stage. On the opposite, the vector lengths can be measured almost exactly by precise and affordable laser instruments. As a result of the inexact azimuths, some error (called displacement error) occurs. The main task in the current paper will be to recalculate the polygon angles without changing the distances and, as a result, to make the displacement error zero or practically negligible. The recalculated polygon can be used then to put exactly the elements in the final stage of construction.

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Memory Effects on a Resonate-And-Fire Neuron Model Subject to Ornstein-Uhlenbeck Noise

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The output of a neuron in neural circuits (*e.g.*, cortical neurons) is typically driven by thousands of synaptic inputs that act as a source of colored noise and make neuronal responses highly variable. Since the generally approved 4D Hodgkin-Huxley neuron model is rather complex, stochastic versions of reduced models, which capture the essence of the electrical activity of a generic neuron, are more interesting from both analytical and computational points of view. Moreover, an understanding of the neuronal mechanisms, guided by reduced models, could be useful in the engineering of artificial neuronal devices, designed to reproduce real biological features [1]. One of the possibilities for modeling reduced dynamics of neuronal activity can be formulated in the framework of the generalized Langevin equation (GLE), where a nonlocal dissipative term (a memory kernel) and colored noise reflect finite-time effects. Recently, a generalization of the stochastic resonate-and-fire (RF) model [2] to the context of GLE driven by Ornstein-Uhlenbeck noise, was subjected to extensive numerical investigation in [1]. Motivated by the results obtained by a computer simulation in Ref. [1], we consider analytically an RF model similar to the one presented in [1]. The effect of temporally correlated random neuronal input is modeled as an Ornstein-Uhlenbeck noise in the GLE with an exponentially decaying memory kernel. We provide, in the noise-induced spiking regime of the neuron, exact analytical formulas for the dependence of statistical characteristics of the output spike train, such as the probability distribution of the interspike interval (ISIs) and the survival probability, on the parameters of the input stimulus. Particularly, on the basis of these exact expressions, we have established sufficient conditions for the occurrence of memory-time induced transitions between unimodal and multimodal structures of the ISI density, and a critical memory time which marks a dynamical transition in the behaviour of the system.

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Numerical Investigation of Biofilm Growth in Microgravity

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In this work, Discontinuous Galerkin Finite Element simulations are performed to investigate the effect of microgravity on biofilm growth. We examine the case of biofilm suspended in a quiescent aqueous oxygen solution contained in a rectangular tank. The problem is modeled by a coupled system of nonlinear partial differential equations in two spatial dimensions. The nutrient and biofilm concentrations are computed and results in microgravity are compared to simulations in 1g and experimental results found in the literature. In addition, a preliminary quantitative relationship between the biofilm concentration and the gravity field intensity is derived.

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Building a Platform to Collect Crowdsensing Data – Preliminary Considerations

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Recent years have seen growing interest in collecting and processing Big Data. In this context, two, somewhat contradictory, trends have emerged: (1) Growing popularity of crowdsourcing type mechanisms, indicative of willingness to share information and resources; (2) Development of vendor-specific platforms that collect data, for example, from sensors "carried by humans." This data remains locked, for instance, by the producers of cellphones and/or fitness bands. Given this limitation of data accessibility, enormous potential for knowledge discovery is lost due to partitioning of data into closed data silos, which are, most often, incompatible, to which Big Data mining techniques cannot be applied.

To counter this trend, the aim of our work is to develop a system that will allow voluntary sharing of sensor data, that is, voluntary participation in crowdsensing. This system will make it possible for persons to opt into a data-sharing platform and decide which data, with whom and when to share.

The proposed system utilises a rule-based multiagent approach, to instructing sensors when to make readings and how to, if necessary, preprocess these readings before sharing the data. In order to facilitate preprocessing, software agents have been placed within sensing devices (for example, cell phones). The proposed presentation will summarise the current state of the system.

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Combining Semantic Technologies with a Content-Based Image Retrieval System – Preliminary Considerations

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Nowadays, we face a huge growth of information that can be found on the Internet. Let us assume that images collected from the internet have been processed using image processing software. As a result a relational database has been created, which contains results of image processing, constituting a Content-based Image Retrieval System (CBIRS). To make such system useful for non-specialists an appropriate interface has to be developed. Such interface can be further augmented by adding semantic technologies, allowing formulation of queries based on a domain ontology.

In the presentation we will illustrate how we were able to combine a domain ontology for "residential real estate" with a relational database of processed images of "family houses." Here, as a result of image processing, for each house, a number of constitutive architectural elements have been identified and represented in the building ontology. In the resulting hybrid system, semantic queries can be applied by non-specialists to search collected and processed images.

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On Resource-Efficient Algorithm for Nonlinear Systems Approximate Reachability Set Construction

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The research considers the numerical solution method of the reachability set construction problem for nonlinear dynamical system in n-dimensional Euclidean space. The study deals with the dynamical system on the finite time interval, which is described by differential equation satisfying a set of defined conditions. The existing step-by-step pixel methods are based on the time interval sampling and applying the step-by-step reachability set constructing procedure to every time moment in mesh. These methods allow us to solve the approximate reachability set constructing problem for the complex nonlinear systems, which do not have analytical solutions. However, applying these methods causes a sharp increase of number of points used for reachability set constructing on the next step of time mesh. This results in increase of calculation time as well as lack of computing device memory. To reduce the calculation time and satisfy the existing constraints of used device memory, we developed two set filtration algorithms based on various ways of picking the points, which are used on the next step of reachability set constructing algorithm. Moreover, the computations are moved from CPU to the CUDA based GPU, which allows us to run computations with the hundreds of parallel threads. In this research, we provide the comparison of described algorithms in several parameters, such as precision, computation time, and the amount of computation resources necessary for the algorithm operation.

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Some Types of Convergence of Sequences of Functions

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In this work we introduce the statistical convergence to a compact set using convergence theory to a set and a sequence of functions. This new notion is close to the set of limit points and the set of cluster points. The set of statistical cluster points turn out to be very useful and interesting tool in turnpike theory to study optimal paths. It has also been discussed in convex or non-convex optimal control problems in discrete systems. In classical theory of convergence, statistical convergence has a special place and these are also active research area. We give some properties of the compact set of statistical uniform cluster functions and we also investigate some inclusion relations for the set of statistical uniform cluster functions.

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Open Formula of Runge-Kutta Method for Solving the Initial Value Problem of the Ordinary Differential Equation

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In this paper open type formulae of both types for the Runge-Kutta method for solving the initial value problem of the ordinary differential equations. One example will be illustrated.

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Mathematical Modeling of the Wave Surface of Non-Isothermal Liquid Film

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The relevance of the study of liquid films associated with the implementation of their flow in the heat-mass-transfer devices. Liquid film (a thin layer of a viscous fluid) flowing in the heat-mass-transfer apparatus at moderate Reynolds numbers is investigated. A system of Navier-Stokes equations and continuity equations with boundary conditions for a non-isothermal liquid film is presented. Derived dispersion equation whose coefficients involve the parameters: surface tension, surface viscosity, Marangoni parameter, shear stress. The nonlinear differential equation for the deflection of the free surface of the film is presented. Increment and phase velocity of vertical liquid film for water under free drainage of liquid films and with account for the non uniformity of surface tension are calculated. The transition from differential equations for the free surface equation in finite differences is made. The results of computer simulation of nonlinear processes: wave characteristics and the state of the surface section of gas-liquid are present. The novelty is the nonlinear mathematical model of free surface liquid film. The coefficients of the equation include parameters of the surface viscosity, Marangoni.

Keywords: Liquid film, increment, phase velocity, nonlinear mathematical model, the free surface

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The Generalized Riemann Solvers and GRP Schemes for Compressible Fluid System

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The generalized Riemann problems (GRP) for nonlinear hyperbolic systems of balance laws are well-known and can be formulated as follows: Given initial data which are smooth on two sides of a discontinuity, determine the time evolution of the solution near the discontinuity. In particular, the GRP of $(k+1)$th order high-resolution is based on analytical evaluation of time derivatives up to kth order. While the classical Riemann problem serves as a primitive 'building block' in the construction of many numerical schemes (most notably Godunov scheme), the analytic study of GRP leads to an array of 'GRP schemes', which extends the Godunov scheme.

In this report, we present the recent work about GRP solvers for the compressible fluid system [1]. For the one-dimensional GRP, we demonstrate the full analytical solvers up to third order, including both the nonsonic and sonic cases. The acoustic approximate solvers are also derived. Our method relies heavily on the new approach to resolve the rarefaction waves. Indeed, as a main technical step, the linear ODE systems for the 'evolution' of the 'characteristic derivatives' of the generalized Riemann invariants are derived. Then we show, via numerical tests, that the analytical solvers are accurate even for solutions with strong discontinuities. Finally, the GRP solvers are adapted to construct high-order GRP schemes for compressible flows in both one and two dimensional spaces. Numerical examples are presented to demonstrate the performance of the resulting schemes.

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Planning Outstanding Reserves in General Insurance

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Each insurance company have to ensure its solvency through presentation of accounts for its own reserves in the start of the year. Usually the task of the actuary is to estimate the state of the company on an annual basis and the expectation of the status of the company for a future period. One of the major problem when calculating the liabilities of the incurred claims, is related to the delay of payments. Object of consideration in the present note are the outstanding claim reserves, which are set aside to cover claims, occurred before the date of the annual account, but still not paid, and related with them expenses. There may be different reasons for the delay of claims settlement. For example, continuation the process of the liquidation of the damage waiting for necessary documents or the presence of controversial cases whose permission takes time, *etc.* Thus the claims, which determine the outstanding reserves could be divided in the following types: claims, which are reported, but not settled (RBNS); claims, which are incurred but not reported (IBNR); claims, whose case is finished, but it is possible to be reopened. When calculating the reserves for IBNR claims, most widely used is the Chain-ladder method and its modification presented by the Bornhuetter-Ferguson method. For modelling the outstanding claims, the available data should be presented in so called run-off triangle, which underlies in the basis of such methods. The present work provides a review of the algorithm for calculating insurance outstanding claim reserves according to the Chain-ladder method. Using available data for claims related to liability of drivers, registered in Bulgaria an example is constructed to illustrate the methodology of the Chain-Ladder method. Back-testing approach is used for validating the results.

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Interspike Interval Distribution of a Resonant-And-Fire Neuron Model Driven by a Mittag-Leffler Noise

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It is well known that noise can have a significant impact on the response dynamics of nonlinear systems. Until relatively recently, most of the stochastic models of neurons studied analytically in neuroscience have focused on the influence of Gaussian white noise. However, the real effective random influence of other neurons on the synaptic inputs of cortical neurons should be considered as a colored noise process with a characteristic memory time. Moreover, recent investigations indicate that some structures of the complex neural networks of the brain have a fractal character [1], which may cause a power-law memory in the spike generation process of neurons. One of the possibilities for modeling the reduced dynamics of resonate-and-fire (RF) neurons [2] reflecting the possible memory effects and the influence of a colored noise can be formulated in the framework of the generalized Langevin equation (GLE), see Ref. [3]. Motivated by the reasons presented above, we consider a RF model based on the dynamics of a GLE subjected to a power-law memory kernel with a finite memory time. The effect of temporally correlated random neuronal input is modeled as a Mittag-Leffler noise; both the internal and external noises are considered. Using a first-passage-time formulation, the exact expression for the output interspike interval (ISI) density and its dependence on input parameters, especially on the memory time, is analyzed. Particularly, in the case of external noise, it is shown that at intermediate values of the memory exponent the survival probability (the probability that spikes are not generated) is significantly enhanced in comparison with the cases of strong and low memory, which causes a resonance-like suppression of the ISI distribution *vs* the memory exponent.

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Influence of Noise on Decay Predictions in Standing Trees

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In the present study, the stochastic process, termed the Vasicek, have been implemented to analyze propagation velocity data of stress wave that diffuses through the wood of decayed and sound stems in standing trees. Stochastic processes offer the possibility of capturing important distributional differences between stress wave velocity datasets of decayed and sound trees and make the detecting rule flexible to internal decay in standing trees. It is shown that the Vasicek type stochastic process is a possible candidate for statistical classification of waveforms. Rigorous quantile regression analysis is based on the conditional probability density functions whose are fitted for stress wave velocity datasets of sound and decayed trees. New developed detector rules derived from stochastic differential equations may be used as an initial tool for detecting internal decay in standing trees.

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Analysis of The Stochastically Forced Invariant Manifolds of Dynamic Systems

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Many nonlinear dynamical phenomena are related to the chain of bifurcations: a stationary regime – periodic regime – quasiperiodic regime – chaos. Each regime is associated with the specific type of attractors (*e.g.*, equilibrium, limit cycle, torus, strange attractor). An invariant manifold can serve as a convenient mathematical model for the elaboration of the general methods for the analysis of these attractors. An important direction of the scientific research is connected with the study of the impact of random disturbances. In the present talk, we consider the randomly forced invariant manifolds of nonlinear dynamic systems. To study the dispersion

of random states near unforced deterministic attractors, we suggest a new semi-analytical method based on the stochastic sensitivity functions. A corresponding mathematical theory is presented and discussed. Constructive applications of this theory to the analysis of complex oscillatory regimes are demonstrated.

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Modeling and Stochastic Analysis of Dynamic Mechanisms of The Perception

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Modern studies in physiology and cognitive neuroscience consider a noise as an important constructive factor of the brain functionality. Under the adequate noise, the brain can rapidly access different ordered states, provide decision-making by preventing deadlocks. Bistable dynamic models are often used for the study of the underlying mechanisms of the visual perception. In the present talk, we consider a bistable energy model subject to both additive and parametric noise. Using the catastrophe theory formalism and stochastic sensitivity functions technique, we analyze a response of the equilibria to noise, and study noise-induced transitions between equilibria. Stochastic bifurcations connected with the suppression of oscillations by parametric noises are discussed.

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Numerical Analysis of Fan-Shaped Waves in Deep Tectonic Faults

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A new mathematical model is proposed for studying the propagation of fan waves with formation of tectonic faults in hard rock massifs at depths of seismic activity. A rock is considered as a multi-block medium consisting of elastic blocks interacting through compliant interlayers under conditions of strong hydrostatic compression. The tectonic fault is a narrow extended zone, filled with solid or deformable fragments - short plates that form a fan-shaped structure (the so-called domino-structure) and perform rotational motion under the action of tangential stresses caused by external natural or technogenic factors. The initial stress-strain state around the fan is given on the basis of a numerical solution of the static problem for a plane with a cut, on which nonlinear boundary conditions of interaction of elastic blocks through the fan are posed. Displacements and stresses are represented in the integral form as a superposition of edge dislocations with an unknown function of distribution of the Burgers vector.

Boundary conditions of the interaction between blocks under solution of the dynamic problem are formulated for velocities and stresses taking into account rotational motion of the plates in a fan. Parallel algorithm for numerical realization of the model is developed, in which parallelization of computations is carried out at the stages of splitting method with respect to spatial variables. A special stage of algorithm is the solution of variational inequality that describes contact interaction of plates through a fan, using the original algorithm for correcting the angular velocity. Based on the developed computer program for multiprocessor systems of cluster architecture, a series of calculations were performed. Dependence of fan velocity on tangential stress, acting on the fault banks, was analyzed.

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On Mathematical Modeling of the Deformation of Fiber-Reinforced Composites Taking into Account Different Resistance to Tension and Compression

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Fibrous composite is a structurally heterogeneous material consisting of two or more components, among which one can distinguish reinforcing elements and a matrix providing joint work of these elements. Traditional mathematical models, used in engineering under calculating fibrous composites for strength, do not take into account the different resistance of a material to tension and compression. The simplest example of a stress-strain state with alternating deformations is the state of longitudinal bending, in which the fibers located on opposite sides of the neutral surface of a plate are stretched or compressed, respectively. We show, that in the problem of three-point bending of a composite plate, which fibers break under compression with a corresponding decrease in the effective modulus of elasticity of a material, the error in determining the deflection on the basis of traditional model can be from one up to several tens of percents, depending on the degree of reinforcement.

The constitutive equations of fibrous composites having different resistance to tension and compression with isotropic matrices and with hetero-modular matrices, whose elastic characteristics change with a change in the sign of strains, are constructed on the basis of a generalized rheological method. Tensors of elastic moduli are determined using the Reuss–Voigt rule of mixtures. Projectors onto the sets of admissible stresses and strains in the tensor spaces are constructed, which are necessary for numerical realization of the constitutive equations in dynamic and static problems of structural elements from fiber-reinforced composites under altern-ating loads. The questions of application of the proposed models to simulation of the deformation of composites with porous matrices taking into account the collapse of pores are considered.

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A Robust PCA-Based Algorithm for Metagenomic Biomarker Detection

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Recent progress in sequencing technologies have enabled the characterization of the microbial communities inhabiting the human body. In the same time, recently, many metagenomic studies have suggested using microbial taxa as potential biomarkers for certain diseases. There are two major challenges associated with the design and the performance assessment of biomarker detection algorithms. The first challenge is to assess the reproducibility performance of an algorithm and to design algorithms that exhibit consistent results. The second challenge is due to the fact that microbial communities exhibit complex interactions among their constituent members. This renders the univariate algorithms that treat these features individually ineffective in capturing such complex dependencies. Although many approaches were proposed to identify metagenomic biomarkers, none of them addresses specifically the reproducibility of the results.

In this work, we propose a new metric to determine the reproducibility performance of the biomarker detection algorithms. Such a metric is referred to as the normalized common-feature consistency. To capture the reproducibility performance probabilistically, we propose also a framework to estimate its distribution based on random resampling. In this work, we also propose a robust principal component analysis (RPCA) based algorithm for biomarker detection. This is achieved by modeling the metagenomic data matrix as a superposition of a low-rank matrix depicting the abundance levels of the irrelevant bacteria and a sparse matrix capturing the abundance levels of the bacteria that are differentially abundant between different phenotypes. Comprehensive comparisons of RPCA with the state-of-the-art algorithms on two realistic datasets are conducted. Results show that RPCA consistently outperforms the other algorithms in terms of classification accuracy and reproducibility performance.

The proposed RPCA-based biomarker detection algorithm provides a high reproducibility performance irrespective of the complexity of the encountered dataset and the number of selected biomarkers. Also, the proposed RPCA-based algorithm could select biomarkers with quite high discriminative accuracy. Thus, RPCA appears to represent a consistent and accurate tool for selecting taxanomical biomarkers for different microbial populations.

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Resource Allocation in Cognitive Communication Systems via Convex Optimization

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The problem of maximizing the total throughput of secondary (cognitive) users in an orthogonal frequency division multiple access (OFDMA) cognitive radio network is addressed in this work. In addition, the power of cognitive users is controlled to keep the interference introduced to primary users under control, which gives rise to a non-convex mixed integer non-linear programming (MINLP) optimization problem. It is illustrated that the original non-convex MINLP formulation admits a special structure and the optimal solution can be achieved efficiently using a standard convex optimization method under quite general and practical assumptions.

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Optimization of Layouts of Low-Rise Residential Buildings

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The problem of the optimal layout of low-rise residential building is considered. Each apartment must be no less than the corresponding apartment from the proposed list. Also all requests must be made and excess of the total square over of the total square of apartment from the list must be minimized. The difference in the squares formed due to with the discreteness of distances between bearing walls and a number of other technological limitations. It shown, that this problem is NP-hard. Built a linear-integer model and conducted her qualitative analysis. It proposed an accurate algorithm for the solution of problems of small dimension. As well, authors developed a heuristic algorithm. The computational experiment was conducted which confirming the efficiency of the proposed approach. Practical recommendations on the use the proposed algorithms are given.

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Hermitian Finite Elements for the Approximation Near a Curvilinear Boundary and in Three-Dimensional Domains

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Earlier, the authors demonstrated that Hermitian finite elements are more efficient in comparison with Lagrangian ones of the same polynomial complexity in the two-dimensional case ("New Hermite Finite Elements on Rectangles," *AIP CP* **1773**, 2016, paper 100005, 7p). Namely, for a second-order elliptic equation for the same accuracy of an approximate solution in the Galerkin method, the number of unknowns and equations of the discrete system of the finite element method for Hermitian elements is less than that for Lagrangian ones of the same degree. Moreover, for the elements whose degrees of freedom involve second-order derivatives, a further reduction of the number of unknowns and equations is demonstrated by invoking the linear relation at certain points of the collocation between the derivatives directly from the differential equation to be solved.

In the proposed report, both properties are justified for the three-dimensional Hermitian finite elements on rectangular cuboids, where the gain in efficiency in comparison with Lagrangian ones is still more significant. In addition, new Hermitian finite elements are presented for approximating a solution near a curvilinear boundary with a nontrivial proof of their accuracy in the Galerkin method. The presented theoretical constructions are confirmed by numerical experiments.

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The Research of Controllable Motion in a Neighborhood of Collinear Libration Point by Conservative Methods

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Currently, several modern space projects are related to the transportation of small celestial bodies on orbit in near-Earth space. A small celestial body can be used to counteract the collision with Earth of a hazard asteroid. The main problems here are optimization of energy costs and safety. In this paper we use the applicable approximation of equations of restricted three-body problem, so-called Hill's model as a mathematical model of motion. We propose the control law for the stabilization of controllable motion of a small celestial body in a vicinity of collinear libration point, which ensures rather low energy costs. The results of numerical modeling, that show the effectiveness of the proposed control law are presented. Numerical simulation is realized both with the help of classical computational procedures and with the help of special conservative methods that take into account the Hamiltonian form of the equations of motion.

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Algorithm for Solving of Two-Level Hierarchical Minimax Program Control Problem of Final State the Regional Socio-Economic System in the Presence of Risks

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This paper discusses a discrete-time dynamical system consisting of a set a controllable objects (region and forming it municipalities). The dynamics each of these is described by the corresponding vector linear discrete-time recurrent vector equations and its control system consist from two levels: basic (control level I) that is dominating and subordinate level (control level II). Both levels have different criterions of functioning and united a priori by determined informational and control connections defined in advance. In this paper we study the problem of optimization of guaranteed result for program control the final state of regional socio-economic system in the presence of risks. For this problem we propose an economic-mathematical model of two-level hierarchical minimax program control the final state of regional socio-economic system in the presence of risks. In this paper for solving of the investigated problem is proposed the algorithm that has a form of a recurrent procedure of solving a linear programming and a finite optimization problems.

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The Research of the Coupled Orbital-Attitude Controlled Motion of Celestial Body in the Neighborhood of the Collinear Libration Point L_1

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This paper considers the motion of a celestial body within the restricted three-body problem of the Sun-Earth system. The equations of controlled coupled attitude-orbit motion in the neighborhood of collinear libration point L_1 are investigated. The translational orbital motion of a celestial body is described using Hill's equations of circular restricted three-body problem of the Sun-Earth system. Rotational orbital motion is described using Euler's dynamic equations and quaternion kinematic equation. We investigate the problems of celestial body motion stability in relative equilibrium positions and stabilization of a celestial body motion with proposed control laws in the neighborhood of collinear libration point L_1. To study stabilization problems, Lyapunov function is constructed in the form of the sum of the kinetic energy of a celestial body and special "kinematics" function of the Rodriguez-Hamiltonian parameters. The numerical modeling of the controlled rotational motion of a celestial body at libration point L_1 is carried out. The numerical characteristics of the control parameters and rotational motion of the celestial body are given.

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Impulsive Control on The Mittag-Leffler Stability of Fractional-Order BAM Neural Network Systems with Time Varying Delays

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The main objective of this research is to provide stability criteria for a fractional-order BAM neural network model with multiple time-varying delays. To extend the application of fractional calculus to impulsive neural network systems, the Mittag-Leffler stability concept and Lyapunov-like functions will be used. The second objective is elaborating of impulsive control strategies to the equilibrium states. This research will serve as a first step to extend the Lyapunov function method to an impulsive BAM neural network system with Caputo fractional-order derivatives and time-varying delays. Developing of stability and control strategies for fractional-order systems would have many applications. It will play an important role in understanding the control power of some instantaneous perturbations and experiences abrupt changes at certain instants which can be used to compensate the deviating trend in fractional biological and artificial BAM neural network models. The goal of that paper is to demonstrate mathematical tools that engineers and neuroscientists can use to determine how neurons in the model are connected to each other. These results can easily be used to design and verify globally stable networks.

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On Flexible Modeling of Generalized Interest Rate

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This paper briefly addresses the zero or negative interest rate environments in different economies worldwide. Some recent data is provided and an over view of effects but also of inducements of monetary policy makers to introduce negative interest rates. We provide overview of recently developed models for both positive and negative interest rates.

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Regression Trees Modeling and Forecasting of PM10 Air Pollution in Urban Areas

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Fine particulate matter (PM10) air pollution is a serious problem affecting the health of the population in many Bulgarian cities. As an example, the object of this study is the pollution with PM10 of the town of Pleven, Northern Bulgaria. The measured concentrations of this air pollutant for this city consistently exceeded the permissible limits set by European and national legislation. Based on data for the last 6 years (2011-2016), the analysis shows that this applies both to the

daily limit of 50 micrograms per cubic meter and the allowable number of daily concentration exceedances to 35 per year. Also, the average annual concentration of PM10 exceeded the prescribed norm of no more than 40 micrograms per cubic meter. The aim of this work is to build high performance mathematical models for effective prediction and forecasting the level of PM10 pollution. The study was conducted with the powerful flexible data mining technique Classification and Regression Trees (CART). The values of PM10 were fitted with respect to meteorological data such as maximum and minimum air temperature, relative humidity, wind speed and direction and others. As a result the obtained CART models demonstrate high predictive ability and fit the actual data with up to 80%. The best models were applied for forecasting the level pollution for 2 days ahead. An interpretation of the modeling results is presented.

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Improved Estimates for Thermal Fluid Equations

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We consider two hydrodynamic model problems (one incompressible and one compressible) with three dimensional fluid flow on the torus and temperature-dependent viscosity and conductivity. The ambient heat for the fluid is transported by the flow and fed by the local energy dissipation, modeling the transfer of kinetic energy into thermal energy through fluid friction. Both the viscosity and conductivity grow with the local temperature. We prove a strong a priori bound on the enstrophy of the velocity weighed against the temperature for initial data of arbitrary size, requiring only that the conductivity be proportionately larger than the viscosity (and, in the incompressible case, a bound on the temperature as a Muckenhoupt weight).

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A Latently Infected Cell Inclusive Model for HIV Treatment with Time-Varying Antiretroviral Therapy

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We present a mathematical model to investigate theoretically and numerically the effect of latently infected cells in the presence of immune effectors in modeling HIV pathogenesis. Additionally, by introducing drug therapy, we assess the effect of treatments consisting of a combination of several antiretroviral drugs. A periodic model of bang-bang type and a pharmacokinetic model are employed to estimate the drug efficacies. We study the effectiveness of the treatment with respect to the time when it is introduced exploring whether immediate antiretroviral therapy can reduce HIV infection of resting CD4 T-cells. We also investigate numerically how time-varying drug efficacy due to drug dosing regimen and/or suboptimal adherence affects the antiviral response and the emergence of drug resistance.

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On Solving Constrained Wave Equations

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Important mathematical models involve wave equations supplemented by constraints on infinite domains. In general, for the pure Cauchy problem one can prove that the constraints are preserved by the evolution. That is, the solution satisfies the constraints for all time whenever the initial data does (*e.g.*, Maxwell's equations and Einstein's field equations in various hyperbolic formulations). Frequently, the numerical solutions to such evolution problems are computed on artificial space cutoffs because of the necessary boundedness of computational domains. Therefore, well-posed boundary conditions are needed at the artificial boundaries. Moreover, these boundary conditions have to be chosen in such a way that the numerical solution on the cutoff region approximates as best as possible the solution of the original problem on infinite domain, and this includes the preservation of constraints. In this talk, I will present a few ideas and techniques for finding constraint preserving boundary conditions for a large class of constrained wave

equations. Then, I will talk about applications of the theoretical framework to certain models (including Einstein's field equations).

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Investigation of The Numerical Differentiation Formulas of Functions with Large Gradients

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Solution of a singularly perturbed problem corresponds to a function with large gradients. Therefore the question of interpolation and numerical differentiation of such functions is actual.

For interpolation of functions it is widely applied the interpolation based on a Lagrange polynomials on uniform mesh. However, it is known that the usage of such interpolation for the function with large gradients leads to estimates that are not uniform with respect to the perturbation parameter, and therefore to the errors of order of $O(1)$. To obtain the estimates that are uniform with respect to the perturbation parameter we can use the polynomial interpolation on the Shishkin mesh as shown in [1] or we can construct on uniform mesh the interpolation formula that is exact on the boundary layer components as shown in [2].

In this paper the numerical differentiation formulas for functions with large gradients based on the interpolation formulas constructing in [2] are investigated. The formulas for the first and the second derivatives of the function with number of nodes for interpolation two and three are considered. The uniform with respect to the perturbation parameter estimates of error are obtained in the particular cases. Some numerical results validating the theoretical estimates are discussed.

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Efficient Stochastic Approaches for Sensitivity Studies of an Eulerian Large-Scale Air Pollution Model

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Reliability of large-scale mathematical models is an important issue when such models are used to support decision makers. Sensitivity analysis of model outputs to variation or natural uncertainties of model inputs is crucial for improving the reliability of mathematical models. A comprehensive experimental study of Monte Carlo (MC) algorithms based on Sobol sequences for multidimensional numerical integration has been done. A comparison with Latin hypercube sampling and a particular quasi-MC lattice rule based on generalized Fibonacci numbers has been presented. The algorithms have been successfully applied to compute global Sobol sensitivity measures corresponding to the influence of several input parameters (six chemical rate reactions and four different groups of pollutants) on the concentrations of important air pollutants. The concentration values have been generated by the Unified Danish Eulerian Model. The sensitivity study has been done for the areas of several European cities with different geographical locations. The numerical tests show that the stochastic algorithms under consideration are efficient for multidimensional integration and especially for computing small by value sensitivity indices. It is a crucial element since even small indices may be important to be estimated in order to achieve a more accurate distribution of inputs influence and a more reliable interpretation of the mathematical model results.

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Computational Cardiology

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Sudden cardiac death (SCD) from arrhythmias is a leading cause of mortality. For patients at high SCD risk, prophylactic insertion of implantable cardioverter-defibrillators (ICDs) reduces mortality. Current approaches to identify patients at risk for arrhythmia are, however, of low sensitivity and specificity, which results in a low rate of appropriate ICD therapy. Here we present a novel non-invasive personalized approach to assess SCD risk in post-infarction patients based on cardiac imaging and computational modeling. We construct personalized 3D computer models of post-infarction hearts from patients' clinical magnetic resonance imaging data. Each heart model incorporates not only myocardial structure, but electrophysiological functions from the sub-cellular to the organ, allowing for representation of electrical instability. In each heart model, we conduct a virtual multi-site delivery of electrical stimuli from ventricular locations at different distances to remodeled tissue so that the patient's heart propensity to develop infarct-related ventricular arrhythmias can be comprehensively evaluated. Simulations are conducted for each virtual heart, probing its propensity to develop infarct-related ventricular arrhythmia. We term this non-invasive SCD risk assessment approach VARP, a virtual-heart arrhythmia risk predictor.

In a proof-of-concept retrospective study, we assessed the predictive capability of the VARP approach as compared to that of other clinical metrics in a cohort of 41 patients. Statistical analysis demonstrated that a positive VARP test was significantly associated with the primary endpoint, with a four-fold higher arrhythmia risk than patients with negative VARP test. Our results also demonstrate that VARP significantly outperformed clinical metrics in predicting future arrhythmic events.

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Spaceborne SAR

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We will discuss some recent (intriguing) findings that pertain to the imaging of the surface of the Earth by spaceborne synthetic aperture radars (SAR). SAR is a coherent imaging technology that uses microwaves for reconstructing the ground reflectivity as a function of spatial dimensions.

Ionospheric turbulence brings an additional dimension into the SAR analysis that accounts for randomness. The overall error now has two components, deterministic and stochastic, that are fundamentally different. The stochastic component becomes larger as the synthetic aperture gets smaller compared to the outer scale of turbulence. Then, why in the ultimate case of very short aperturesit appears that the stochastic error can be completely disregarded?

The Doppler effect in fast time is neglected when using the start-stop approximation. Yet it can make the images prone to distortions, unless special corrections are implemented in the matched filter. Why does it turn out that the imaging regime most susceptible to distortions is the frequency modulated continuous wave (FMCW) SAR that corresponds to low chirp rates?

Radar targets are typically modeled via the first Born approximation.However, the assumption of weak scattering is inconsistent with that of scattering off the surface only, because the latter implies that radar signals do not penetrate under the surface of the target. How can one take into account the proper physics of the target and construct a scattering model that would be free of inconsistencies yet remain linear and thus amenable to SAR wave field inversion?

We will outline approaches to answering those questions and identify the outstanding issues that require further attention.

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Van der Waals Interactions between Axisymmetric Lipid Membranes and Planar Substrate

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In the current article we study the behavior of the van der Waals interaction force between a planar substrate and an axisymmetric bilayer lipid membrane subject to uniform hydrostatic pressure. To do so, the recently suggested "surface integration approach" is used, which can be considered a generalization of the well known and widely used Derjaguin approximation. Two principal cases are considered: when the axis of revolution is parallel and orthogonal to the plane of interaction. The geometry of the membrane is assumed to be described within the framework of the Helfrich spontaneous curvature model. Some specific classes of exact analytical solutions to the corresponding shape equation are considered, with

the components of the respective position vectors being given in terms of elliptic integrals and Jacobi elliptic functions.

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Numerical Solution of Dynamic Problems in Micropolar Plates and Shells on Supercomputers with GPUs

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Mathematical model of micropolar continuum taking into account independent small rotations of particles is applied for the modelling of the behavior of materials with microstructure: composites, granulate, powdery and loose media. To describe micropolar plates and shells approximation approach based on the reduction of the three-dimensional micropolar continuum equations is considered. In the framework of the approximation approach averaging procedure in the thickness direction with linear approximation of displacements and rotations is used. Then the system of two-dimensional equations describing micropolar plates and shells is written in the form of thermodynamically self-consistent system of conservation laws.

In order to solve problem numerically it is essential to perform calculations with the mesh size comparable with the size of a particle of the material. For this reason parallel computational algorithms for high-performance computer systems are required. In this paper numerical algorithms based on the two-cyclic splitting method for the solution of dynamic problems are proposed. Parallelization is performed by the CUDA (Compute Unified Device Architecture) technology. The results of numerical computations of dynamic problems in micropolar plates and cylinder shells are shown.

Acknowledgement. This work was supported by the Russian Foundation for Basic Research (project no. 16-31-00078).

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About Two-Parameter Family of Nonlinear Differential Equations

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We consider a two-parameter family of nonlinear differential equations describing the behavior of a system within the framework of the Ginzburg-Landau theory by means of the order-parameter. We focus on the case in which the order-parameter tends to infinity at one of the boundaries and to minus infinity at the other one. The boundaries of the system are positioned at a finite distance from each other. We give analytic solutions to the corresponding boundary-value problems in terms of Weierstrass and Jacobi elliptic functions.

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Generation of Correlation Matrices

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Correlation and covariance matrices representing the strength of the dependence for each pair of factors involved in a statistical experiment are always positive definite matrices. The article discusses ways to generate such matrices with or without restrictions on their elements and laws of distribution. Special attention is paid to the case when the resulting correlation matrix presents a weak correlation between pairs of factors involved in the experiment.

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Multifactor Estimation of The Ecological Risks Through Numerical Simulation

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The multifactorial problem of estimation of the geoecological risks to environment and social infrastructure from mass technogenic and natural explosions such, are considered. It was shown earlier that impact of the meteofactors complex to the infrasound propagation at specific conditions may leads to multiple increasing the geoecological risks. At the same time there are factors which lead to attenuation of the ecological risks if snow cover, forest and surrounding area relief present.

The paper represents mathematical simulation of the elastic waves propagation from infralowfrequency source taking into account snow cover based on the theory of dynamic poroelasticity. The snow cover is approximated as porous medium, saturated with liquid or air, where the three elastic parameters are expressed via three elastic wave velocities. These velocities are recalculated using the elastic waves velocities via the Biot theory, which are expressed through elastic parameters of the snow. Note that the obtained solutions allow the study the peculiarities of the seismic waves propagation in the liquid or air, which saturating snow cover. In this case, the obtained formulas allow us to simulate the displacement velocity of the porous frame and the saturating fluid in it, as well as the pore pressure and the stress tensor components with given elastic parameters of the medium and the velocities of propagation of transverse and longitudinal waves in a porous medium.

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Numerical Investigation of Sixth Order Boussinesq Equation

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We study the general Boussinesq equation

$$u_{tt} - u_{xx} - \beta_1 u_{ttxx} + \beta_2 u_{xxxx} - \beta_3 u_{xxxxxx} + \beta_4 u_{ttxxxx} + f(u)_{xx} = 0$$

with sixth order dispersion terms.

For the numerical solution of this problem a family of finite difference schemes is constructed. The preservation of the discrete energy and a second order of convergence of the discrete solution to the exact one in the $W^1_{2,h}$ mesh norm are proved.

The schemes have been numerically tested in the cases of quadratic and cubic function f. The numerical experiments show good agreement with the theoretical results.

Acknowledgement. This research is partially supported by the Bulgarian Science Fund under grant DFNI I-02/9.

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The Locally Conservative Semi-Lagrangian Method for Three-Dimensional Advection Problem

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We develop a semi-Lagrangian algorithm for solving the three-dimensional advection problem. It is a model problem for both compressible and incompressible fluids. To compute a numerical solution, we use a uniform time grid and a uniform cubic triangulation in three-dimensional space. We determine a numerical solution as a function that is piecewise constant in cubic sells at each time level. The method is based on the local integral balance equation between two neighboring time levels, which follows from the Gauss-Ostrogradsky theorem. To find a numerical solution at the next time level at the center of a sell, we integrate it over this sell. This produces the left-hand side of the algebraic equation. To get the right-hand side, we consider the integral of the numerical solution at the previous time level, which has already been determined. To obtain the integration domain for this integral, we project this sell onto the previous time level along characteristics and get a curved cuboid. We decompose it into eight subdomains each with eight vertices. To compute the integral over a subdomain, we construct the trilinear transformation of the unit cube into a subdomain with the same eight vertices. Thus, we replace the integration over a curved subdomain by the integration over the unit cube. Then we use a simple cubature rule on the unit cube with the Jacobian of the transformation. This gives the right-hand side for the explicit algebraic equation. The proposed difference scheme is of first-order convergence that is confirmed by computational experiments.

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Shock-induced Microspall in Tin Plate

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When a shock wave produced by a high explosive detonation reaches the free surface of most metals, different phenomena can occur. For example, one or more layers of solid material is produced from the fracture of the accelerated metal, the metals could also be melted on release and a fragmentation process is expected, leading to the expansion of a cloud of fine fragments from the free surface. The detail understanding of these complex damage phenomena in metals is an active area of research in shock physics.

In this paper, we present an experimental investigation of tin samples under HE driven shocks. A 25mm-diameter, 30mm-thick cylinder of HE was point initiated with a detonator centered on the charge. A series of experiments were performed in four kinds of thickness. Diagnostic methods included x-ray radiography and Asay window which can give density and velocity information about the spall layers or the fragments.

Numerical simulations of the experiments were conducted. A thermodynamic consistent Helmholtz free energy combined with a nonequilibrium kinetics of relaxation model was used to describe the dynamic response induced by phase transition in tin. The mathematical model of damage was a form of void growth originally proposed by J.N. Johnson, we introduced the effects of melt fraction and Gibbs energy. The simulated results indicated that tin melt is influenced by the thickness of the samples. For thin sample the melted region was dominated by the reflection of stress wave from the free surface. While for thick sample the melted region was formed by the release of Taylor wave and from side boundary. The simulations also suggested that four modes of fracture occurred in the tin samples. The computed distribution of areal density was improved with the above models and could agree well with experimental records.

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Study on Turbulent Mixing Induced by Rayleigh-Taylor Instability Using The BHR k-S-a-b Model

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The BHR k-S-a-b model is developed based on the BHR model proposed by Besnard *et al.* In the model the transport equations of turbulent kinetic energy, turbulent length scale, mass-weighted turbulent velocity and density-specific-volume covariance are used to describe turbulence. In the present study, the model is used to study the turbulent mixing induced by Rayleigh-Taylor instability (RTI). In the flow induced by RTI turbulence is driven by pressure and density gradients instead of velocity gradient, which is different from the shear flow. The BHR k-S-a-b turbulence mix model has been implemented in the HIME hydro-code. Third-order Runge-Kutta method is used to carry out time integration. Advection terms use high order WENO method to reconstruct fluxes on grid boundaries. Alternate direction implicit (ADI) method is used to solve diffusion terms. On the basis of these, the low Atwood number (At = 0.04) RT experiment and the moderate Atwood number (At = 0.5) RT direct number simulation (DNS) case are simulated by this model. The model results are compared with detailed experiment data and DNS results to prove the validation of the model closure, model constants, numerical algorithm and the implementation of the model used in the paper. It is also found that the model reasonably captures the asymmetry or shift of the peak turbulent energy toward the spike side at the moderate Atwood number.

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Method for Solving a Problem of Nonlinear Heating a Cylindrical Body with the Unknown Initial Temperature

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We consider the problem of heating a cylindrical body with the internal thermal source when the main characteristics of the material such as specific heat, thermal conductivity and material density depend on the temperature at each point of the body. We can control the heat flow and temperature from the surface inside the cylinder, but it is impossible to measure the initial temperature in the entire body. This problem is associated with the measurement challenge of temperature and appears in non-destructive testing, in thermal monitoring of heat treatment and technical diagnostics of equipment operating. The mathematical model of heating is represented as nonlinear parabolic PDE with the unknown initial condition. In this problem, both Dirichlet and Neumann boundary conditions are given and it is required to calculate the temperature values at internal points of the body. To solve this problem, we propose the numerical method based on using of finite-difference equations and a regularization technique. The computational scheme involves solving the problem for each spatial step. As a result, we obtain the temperature function at each internal point of the cylinder beginning from the boundary down to the axis. The application of the regularization technique ensures the stability of the scheme and allows us to simplify significantly the computational procedure. We investigate the stability of the computational scheme and prove the dependence of the stability on discretization steps and error level of the boundary function. To obtain the experimental temperature error estimates, computational experiments were carried out. The computational results are consistent with the theoretical error estimates and confirm the efficiency and reliability of the proposed computational scheme.

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Wave Dynamics of Deformation and Fracture

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Based on the physical principle known as local symmetry, deformation and fracture of solids are formulated comprehensively.

The gist of this formalism is as follows. Linear elastic deformation can be expressed as linear transformation of local volume elements. Here the transformation matrix is the deformation gradient tensor. Since Hooke's law represents stretch or compression, the dynamics associated with this transformation is an orientation preserving mapping. Consequently, the formalism is applicable to an object that experiences stretch or compression in a certain orientation. When different parts of the object undergo stretch or compression in mutually different directions, the dynamics cannot be described by a single transformation matrix at the global level because the transformation is not orientation preserving any more.

A simple and realistic example of such a case is plastic deformation. When an initially elastic object enters the plastic regime due to an external load, a defect generated at a certain location of the object causes the parts surrounding the defect to stretch their own orientations. In this situation, it becomes necessary to use a deformation gradient tensor for each individual part separately. Consequently, at the global level, the deformation gradient tensor becomes coordinate dependent. Since the deformation gradient tensor contains differential operations, it follows that the derivatives must be replaced with covariant derivatives, *i.e.*, a gauge field must be introduced.

The present formalism derives Lagrangian associated with this gauge field, and applying the least action principles, it derives field equations. These field equations describe dynamics of all stages of deformation and fracture on the same physical basis. The general solution to the field equations represents wave dynamics of deformation. Different stages of deformation can be identified by characteristic feature of the wave dynamics.

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Numerical Solutions to an Energy Concentration Problem Associated with the Affine Fourier Transform

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The goal of this talk is to present numerical solutions to a concentration of energy problem associated with the Affine Fourier Transform, which is an integral transform associated with a general inhomogeneous lossless linear mapping in phase-space that depends on six parameters independent of the phase-space coordinates.

Since an explicit, closed form solution seems to be elusive, we will solve the problem numerically. The problem can be reduced to finding the largest eigenvalues and their associated eigenfunctions of two-dimensional integral equations. The numerical solutions are obtained by using the Gaussian quadrature method in two dimensions.

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An Effective 3D MMALE Method for Compressible Fluid Dynamics

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An effective second-order three-dimensional unstructured multi-material arbitrary Lagrangian-Eulerian (MMALE) method was presented for compressible fluid dynamics, which uses Moment of Fluid (MOF) method to reconstruct material interface for immiscible fluids. It is of the explicit time-marching Lagrange plus remap type. Comparing with traditional ALE method, MMALE method permits multi-materials in a single cell, thus has the extra advantage of accurately modeling problems involving severe mesh distortion as well as interface fragmentations and coalitions induced by strong shearing deformation. Because the stencil used in the staggered compatible discretization involves only the nearest neighbor cells and the MOF algorithm does not need information from the neighboring cells, the MMALE method in this paper is suitable for parallel computation while keeps second-order accurate. Several numerical tests on three-dimensional structured and unstructured meshes have proved the accuracy and robustness of the present method.

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Analysis and Modeling of Daily Air Pollutants in The City of Rousse, Bulgaria

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The city of Rousse is situated in the north-eastern part of Bulgaria. The northern boundary of Rousse region goes along the Danube river valley and coincides with the state boundary of the Republic of Bulgaria and the Republic of Romania. The climate of the region of Rousse is temperate continental, characterized by cold winters and dry, warm summers. Spring and autumn are short.

In our previous work we studied information from 40 years period measurements of temperature, air humidity and atmospheric pressure in Rousse region, Bulgaria. It was shown that mean values of the temperature in Rousse region are slightly goes up for last 10 years and they are bigger than the mean temperature for Bulgaria. This could be a proof for climate change in Rousse region of Bulgaria.

The most variable atmospheric parameter is air humidity during the spring seasons. The hardest change of temperature and atmospheric pressure is during January. Temperature has biggest change in January and smallest – in July. Humidity has biggest change in April and smallest – in October. Atmospheric pressure has biggest change in January and smallest – in July. Also it is known that the valleys and places along the rivers are polluted stronger than the plains and slopes. All this in our opinion may be a reason for the increase in average temperatures for the period examined. Air pollution maybe affects temperature, atmospheric pressure and humidity.

This paper is devoted to examine the influence of atmospheric characteristics on air pollution in the Rousse region. It presents a statistical analysis of the level of air pollution in Ruse on data from three monitoring stations in the city. The measurements cover the period from 2015 including up to now.

The results are compared with similar studies conducted for other cities in Bulgaria.

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The Stability of Tubes of Discontinuous Solutions of Dynamical Systems

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The article is dedicated to investigation of nonlinear dynamical systems which applies the generalized effect which is the generalized derivative of functions of bounded variation. Such system contains incorrect operation of multiplication of discontinuous function on the generalized function from the point of view of the theory of the generalized functions. This incorrectness is overcome by the approximation of the generalized functions in the right part of system by the sequence of smooth approximations of the generalized influences by analogy with sequential approach of the theory of the generalized functions. This sequence generates a sequence of smooth solutions. Then limit of a sequence of smooth solutions is considered. If such limit exists it is offered to be used in quality of the decision. The decision is understood as partial point-wise limit of such sequence if a sequence of smooth solutions does not converge. Such partial limits constitute a tube of decisions. Integral inclusions describing such tubes of decisions are given in [1]. The stability of solutions in the case when there exists a unique limit of smooth solutions was considered in [2]. The sufficient conditions are received for stability of tubes of discontinuous solutions.

Acknowledgement. The research was supported by Russian Science Foundation (RSF) (project No.16-11-10146).

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Choosing Internal Degree of Freedom for Damage Formation by Discrete Atomistic Analysis

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In case of the theoretical analysis of materials strength, the experimental data is usually compared with the results of the one-dimensional atomistic study which predicts a higher value of the ratio between the critical stress of fracture and Young modulus. In the discrete atomistic approach the mentioned estimation of the critical stress is obtained under uniform tension of the homogeneous atom chain. The present study demonstrates that the random perturbations of the atom placements in the chain (which correspond to the random variations of the material density gradient) lead to the formation of one or several defects similar to the mode-I cracks. These defects are found due to dynamic computations of atoms' motion from the stretched state with the implied random shifts at the initial instance of computation taking into account the artificial viscosity. The latter allows finding the equilibrium stretched configuration (in certain cases with inhomogeneous atom distribution). It is shown that there exists a critical value of the atom chain elongation, and if it is exceeded, it will result in the defect (damage) formation.

The potential energy value which corresponds to the found occurrence of the defect nucleation is significantly less than the energy required for the homogeneous atom chain breaking. If the chain elongation increases, then the size of the defect grows but the atom spacing in the surrounding parts of the atom chains (which remain the whole) holds unchanged. If the chain length that corresponds to the equilibrium stress-free non-defect state is abruptly changing to a large enough value, then several defects are formed. The number of these defects depends on the reached length of the atom chain (or on the applied energy). The equilibrium stretched configurations with different numbers of defects have different values of the internal energy; and the transition from one state to another is accompanied by the step-wise energy change. The minimal energy portion corresponds to the energy of one defect. The "surface atoms" distribution is naturally obtained during the computation, so the potential energy of the material with a defect also includes the internal surface energy. The non-linear non-convex function of the internal energy of the atomic chain with a defect is obtained. This function depends both on the external deformation applied to the atom chain and the internal degree of freedom which is described by the material density gradient averaged along the chain. In such interpretation the considered material has been adapted for the external loading by minimizing its potential energy and forming the system of defects. The found internal energy function considers the surface energy of the defects and demonstrates that by reaching the critical value of the applied elongation, the homogeneous deformation becomes inefficient from the energetic point of view and the material starts to produce the system of defects.

In order to obtain the above results, the periodic interatomic interaction potential was introduced. This potential takes into account the periodic boundary condition; it is obtained as the sum of infinite convergence series based on the Mie family potentials of the pair-wise interaction. Under the summation we supposed that each atom of the chain interacts with an arbitrary chosen second atom as well as with infinite number of images of this second atom placed in the images of the chain. These images are periodically repeated along the coordinate axis. The main section of the atom chain may contain any number of atoms. The periodic potential which describes the interaction of atoms from two parallel periodic atom chains was also built in the work. This potential makes it possible to study the quasi-two-dimensional periodic atom structures and formation of quasi-two-dimensional defects corresponding to the mode-I cracks.

Acknowledgement. The work was supported by the Russian Science Foundation (project No. 17-19-01292).

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Material Rotation and Rate-Type Finite-Strain Elasticity for Two-Dimensional Oblique Medium

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The problem of motion decomposition onto rigid and deformable parts is well known in the nonlinear continuum mechanics of solids from the earlier works of S. Zaremba and G. Jaumann. This problem partially coincides with defining the indifferent corotational rate. Recognizing rigid motion of the material within finite-strain deformations is the only way to write anisotropic elastic law with the changeless elastic moduli. The measures of rotation often being exploiting such as orthogonal tensor R from the polar decomposition of the deformational gradient F and its (polar) spin tensor or the vorticity tensor W, or the logarithmic spin tensor (introduced by H. Xiao, O. Bruhns, A. Meyers) have no relation to the anisotropy axes rotation. The latter is supposed to be associated only with the material symmetry axes motion and has to be defined in a special manner. The approach of recognizing the material rotation considered below requires preserving the type of material symmetry under deformation. All single-crystals being deformed without phase transitions, semi-crystalline polymers and some composites are appropriate instances. For the simplicity of the material spin introduction a general two-dimensional (oblique) medium and two-dimensional deformations are considered.

Definition: the material frame rotation tensor in any chosen material point is a proper orthogonal second rank tensor connecting the actual, but not deformed material symmetry axes (obtained in the actual configuration by releasing a small material neighborhood of the point considered) with the initial ones.

The introduced tensor explains the rotation of the vector frame which coincides with the material symmetry axes in the initial configuration of the material volume and uniquely corresponds to these axes in the actual configuration although it does not exactly coincide with them in the deformed state. Suppose that in the releasing process the material volume reaches its final reloaded state corresponding to the minimal summary path of all material points (across the immovable mass center of the volume) from the deformed configuration into the one released. In the considered case the unit rotation axis n is fixed and orthogonal to the plane of vectors. And then the exact solution of the rotation tensor of material axes is found. In the case of the tetragonal (and also isotropic) media, the introduced material rotation tensor coincides with the rotation tensor from the polar decomposition of deformation gradient. For the case of small displacements the small material rotation differs from the small rotation tensor and depends on the ratio of material axes parameters and the angle between material axes. It is shown that material axes rotation under the two-dimensional deformation along the elliptic closed path is smooth and evidently depends on the initial orientation I_0 in contrast with the orientation of the eigenvectors of left stretch tensor from the polar decomposition of the deformation gradient.

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